PM book: new results

Phase Method:
•Introduces the PM, a simple and effective new method for determining eigenvalues and wave function with some advantages over existing standard methods. The PM is robust, works well for nearly any 1D or 2D/3D central potential, whether smooth or rough or discontinuous or singular. PM also is applicable to solving Helmholtz equation.
•Introduces the "Phase Plot" which allows very fast calculation of the integrated density of states for a given energy
•Explanation of phase plot dynamics by rewriting Schrödinger equation as a linear first order equation in the logarithmic derivative
•Proof that solutions are insensitive (apart from overall scale) to initial conditions over the relevant x-range,
as long as the integration is started well below the lower classical turning point.
•Calculation of scattering phase shifts can be done by PM, without incurring the usual Pi phase ambiguities/discontinuities
•Explanation of Levinson's theorem from scattering theory is made clear by Phase Plot
•Numerous detailed examples are given, including the emergence of band structure in finite period wells as more wells are added.
Connection is made with tight-binding approximation. Convolution approach is proposed that generates approximate eigenfunctions within a band, given only a single-well prototype wave function for the band.
•Singular potentials with k<=-2 have no lower bound on the eigenvalues are explored using Phase Plot as a function of lower distance cutoff, with some intriguing scaling behavior

Semiclassical approaches:
for homogeneous (e.g. power law) potentials:
•Derive *exact* scaling relation for eigenvalues of homogeneous potentials with the same k but different energy and length scale parameters and representations of m and hbar.
•Using "mechanical similarity" and BSW quantization, gives two new formulae for approximate eigenvalues in homogeneous (power law) potentials.
•Introduces parameter *nu* which is related to the degree of tunneling into barrier.
•Use of *nu* extends accuracy far beyond that of WKB method, especially for hard potentials (large k) and small n
•Values of nu are computationally estimated, and meaning explained
•Deviation from standard approach is justified as partially compensating for the terms missing from the *exact* WKB gradient expansion
•Procedure is given for estimating the complete spectrum using any three consecutive eigenvalues
•Explains and demonstrates a duality relation between pairs of homogeneous

potentials e.g. harmonic oscillator <==> coulomb potential

for general potentials:
•Reinterpretation of elementary Bohr-Sommerfeld-Wilson quantization: rather than generally providing a poor estimate of eigenvalues, instead provides *upper bound*s to the exact eigenvalues. This can be used to provide eigenvalue intervals for any 1D potential, useful also for 2D/3D central potentials and other separable potentials.