Plots of optimum filter thickness

Introduction

Following are plots of the effective number of counts (Neff) divided by the number of signal counts, as a function of filter edge step ($\Delta\mu x = \mu x_{above\_edge} - \mu x_{below\_edge}$). By "above edge" we mean some energy in the middle of the EXAFS scan range, and "below edge" means the fluorescence energy.

Families of curves are shown for different background to signal ratios increasing in powers of two as 2^n, where n ranges from 0 to 7, i.e. the background to signal ratio takes values B/S= 1,2,4,8,16,32,64,128. The optimum filter thickness also depends on filter quality ($Q=\mu x_{above\_edge}/ \mu x_{below\_edge}$), and fluorescence rejection factor (eta). In addition, the optimum filter thickness is plotted vs n where $B/S=2^n$, e.g. if the experimental background is 32 times the signal (in absence of filter) then n=5. These curves are also fit with a quadratic polynomial and coefficients are presented below.

The results presented below are appropriate for non-energy dispersive detectors such as fluorescence ionization chambers. Filters may also be beneficial when using energy dispersive detectors, for which the value of eta can be taken to zero, because filter refluorescence can be rejected by window discrimination.

Define equations

```
Ns=Ns0 (Exp[-mubelow]+eta(1-Exp[-mubelow]));
Nb=Nb0 (Exp[-muabove]+eta(1-Exp[-muabove]));
mubelow=x/(Q-1);
muabove=x Q/(Q-1);
Neffective=Ns^2/(Ns+Nb);

Neff[x_,A_,q_,Eta_]:=Neffective/.{Q->q,eta->Eta,Ns0->1.,Nb0->A}

Neffcurves[Q_,eta_]:=Plot[{
  Neff[x,1,Q,eta],
  Neff[x,2,Q,eta],
  Neff[x,4,Q,eta],
  Neff[x,8,Q,eta],
  Neff[x,16,Q,eta],
  Neff[x,32,Q,eta],
  Neff[x,64,Q,eta],
  Neff[x,128,Q,eta]},
  {x,0,8},
  AxesLabel->{"Dmux","Neff/Ns"},
  PlotLabel->{"Q,eta=", Q,eta}]

best[A_,Q_,eta_]:=
  FindMinimum[-Neff[x,A,Q,eta],
  {x,3,0,10}][[2]][[1]][[2]]
```

```
bestvalues[Q_,eta_]:=
  Table[{n,best[2^n,Q,eta]},{n,1,7}]

Timing[
  Do[
    {Print["Q=",q,"  Eta=",e];
    Print["Log2[Nb/Ns]   Optimum Delta Mu x:"];
    opt=bestvalues[q,e];
    If[NumberQ[opt[[1,2]]],opt,Drop[opt,1]];
    Print[ColumnForm[opt]];
    Print["Best Fit: (z=delta mu x)"];
    Print[Fit[opt,{1,z,z^2},z]];
    lin="------------------------------";
    Print[lin];
    min=Min[Transpose[opt][[2]]];
    max=Max[Transpose[opt][[2]]];
    ListPlot[opt,PlotJoined->True,
      AxesLabel->{"Log2(Nb/Ns)","opt DMux"},
      PlotLabel->{"Q,eta=", q,e},
      PlotRange->{{0,7},{min,max}}];
    Neffcurves[q,e]},
    {q,3,7},{e,0,.15,.05}]
  ]
```

■ Numerical Results

Following are tables of the optimum filter thickness (Δμx) vs the logarithm (base 2) of the background to signal ratio. For example, if the background to signal ratio is approximately 64 (2^6), and the filter quality is 4, and the fluorescence rejection factor is 0.1, the best filter thickness is 1.875.

```
Q=3  Eta=0
Log2[Nb/Ns]   Optimum Delta Mu x:
{1, 0.693147}
{2, 1.38629}
{3, 2.07944}
{4, 2.77259}
{5, 3.46574}
{6, 4.15888}
{7, 4.85203}
Best Fit: (z=delta mu x)
          -10                          -11  2
7.99273 10    + 0.693147 z + 2.9841 10    z
------------------------------
Q=3  Eta=0.05
Log2[Nb/Ns]   Optimum Delta Mu x:
{1, 0.583115}
{2, 1.03628}
{3, 1.33776}
{4, 1.51446}
{5, 1.61031}
{6, 1.66022}
```

```
{7, 1.68568}
Best Fit: (z=delta mu x)
                                      2
0.148804 + 0.511297 z - 0.042358 z
-------------------------------
Q=3  Eta=0.1
Log2[Nb/Ns]   Optimum Delta Mu x:
{1, 0.510374}
{2, 0.86787}
{3, 1.08576}
{4, 1.20676}
{5, 1.27058}
{6, 1.30335}
{7, 1.31995}
Best Fit: (z=delta mu x)
                                       2
0.190814 + 0.385821 z - 0.0326718 z
-------------------------------
Q=3  Eta=0.15
Log2[Nb/Ns]   Optimum Delta Mu x:
{1, 0.457013}
{2, 0.759039}
{3, 0.935931}
{4, 1.03204}
{5, 1.08218}
{6, 1.10779}
{7, 1.12073}
Best Fit: (z=delta mu x)
                                       2
0.195147 + 0.319703 z - 0.0273072 z
-------------------------------
Q=4  Eta=0
Log2[Nb/Ns]   Optimum Delta Mu x:
{1, 1.38629}
{2, 2.07944}
{3, 2.77259}
{4, 3.46574}
{5, 4.15888}
{6, 4.85203}
{7, 5.54518}
Best Fit: (z=delta mu x)
                                     -10  2
0.693147 + 0.693147 z - 3.6275 10    z
-------------------------------
Q=4  Eta=0.05
Log2[Nb/Ns]   Optimum Delta Mu x:
{1, 1.20095}
{2, 1.64785}
{3, 1.95683}
{4, 2.14534}
{5, 2.25063}
{6, 2.30643}
{7, 2.33518}
```

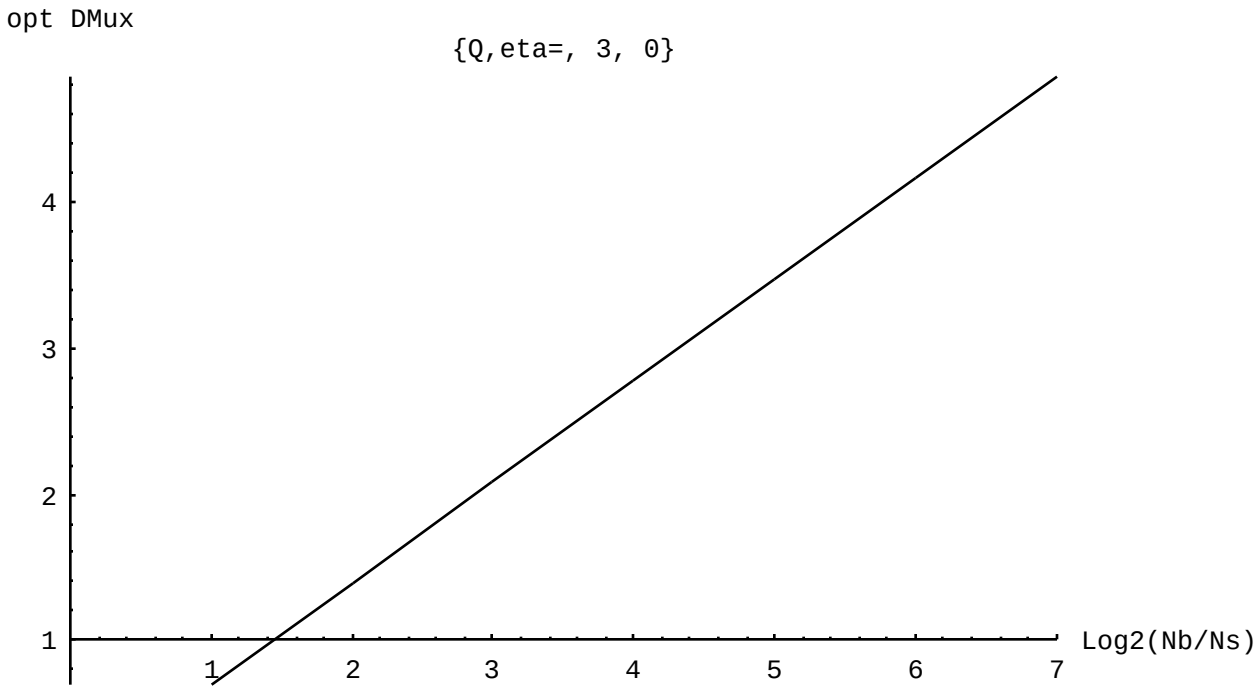
```
Best Fit: (z=delta mu x)
                                        2
0.758063 + 0.514592 z - 0.0419415 z
-------------------------------
Q=4  Eta=0.1
Log2[Nb/Ns]   Optimum Delta Mu x:
{1, 1.07726}
{2, 1.42623}
{3, 1.64645}
{4, 1.77232}
{5, 1.83993}
{6, 1.87502}
{7, 1.8929}
Best Fit: (z=delta mu x)
                                        2
0.756357 + 0.383275 z - 0.0321149 z
-------------------------------
Q=4  Eta=0.15
Log2[Nb/Ns]   Optimum Delta Mu x:
{1, 0.985245}
{2, 1.27836}
{3, 1.45538}
{4, 1.55376}
{5, 1.60579}
{6, 1.63257}
{7, 1.64615}
Best Fit: (z=delta mu x)
                                        2
0.724874 + 0.314969 z - 0.0266855 z
-------------------------------
Q=5  Eta=0
Log2[Nb/Ns]   Optimum Delta Mu x:
{1, 1.79176}
{2, 2.48491}
{3, 3.17805}
{4, 3.8712}
{5, 4.56435}
{6, 5.2575}
{7, 5.95064}
Best Fit: (z=delta mu x)
                                     -8  2
1.09861 + 0.693147 z + 2.84551 10   z
-------------------------------
Q=5  Eta=0.05
Log2[Nb/Ns]   Optimum Delta Mu x:
{1, 1.58423}
{2, 2.03972}
{3, 2.36361}
{4, 2.56683}
{5, 2.68273}
{6, 2.74496}
{7, 2.77725}
Best Fit: (z=delta mu x)
```

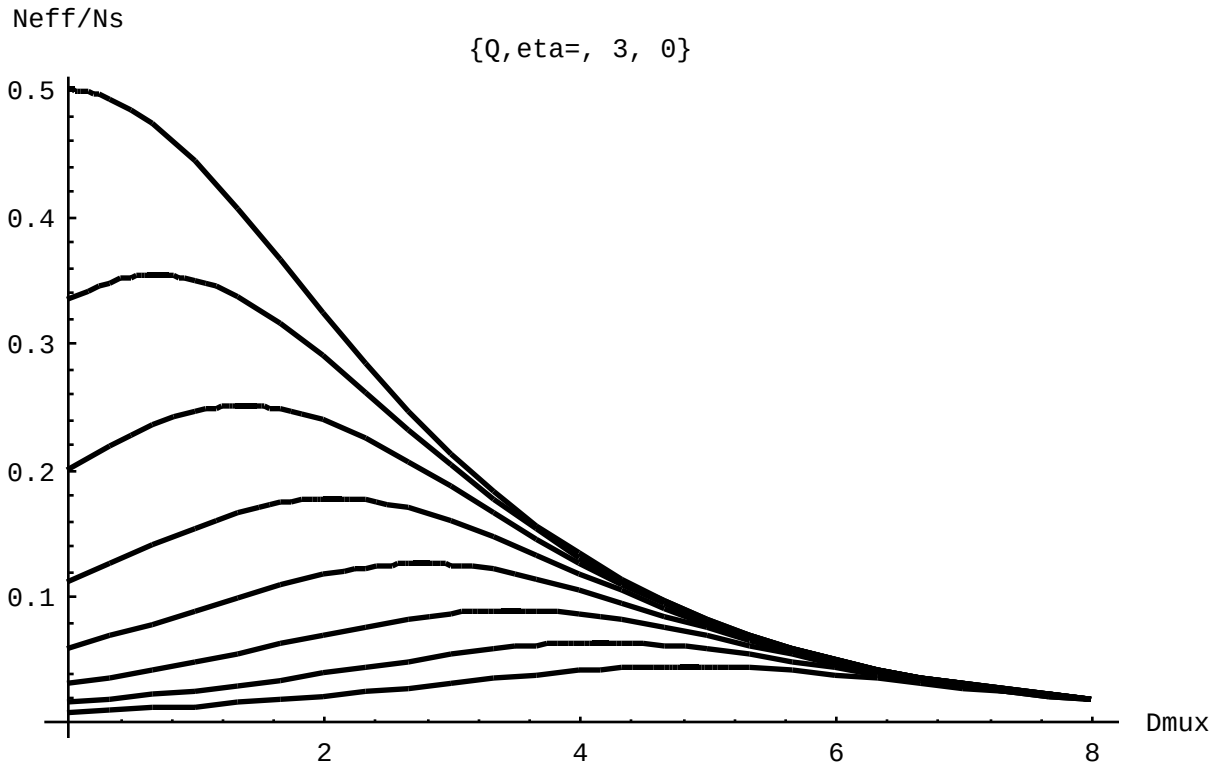
```
                                      2
1.12168 + 0.53235 z - 0.0428443 z
------------------------------
Q=5  Eta=0.1
Log2[Nb/Ns]   Optimum Delta Mu x:
{1, 1.44137}
{2, 1.79693}
{3, 2.02657}
{4, 2.16032}
{5, 2.23306}
{6, 2.27107}
{7, 2.29052}
Best Fit: (z=delta mu x)
                                      2
1.10812 + 0.39515 z - 0.0328659 z
------------------------------
Q=5  Eta=0.15
Log2[Nb/Ns]   Optimum Delta Mu x:
{1, 1.33306}
{2, 1.6309}
{3, 1.81422}
{4, 1.91754}
{5, 1.97266}
{6, 2.00116}
{7, 2.01566}
Best Fit: (z=delta mu x)
                                       2
1.06446 + 0.323069 z - 0.0272285 z
------------------------------
Q=6  Eta=0
Log2[Nb/Ns]   Optimum Delta Mu x:
{1, 2.07944}
{2, 2.77259}
{3, 3.46574}
{4, 4.15888}
{5, 4.85203}
{6, 5.54518}
{7, 6.23832}
Best Fit: (z=delta mu x)
                                     -8  2
1.38629 + 0.693147 z + 2.18528 10   z
------------------------------
Q=6  Eta=0.05
Log2[Nb/Ns]   Optimum Delta Mu x:
{1, 1.86437}
{2, 2.32878}
{3, 2.66577}
{4, 2.88162}
{5, 3.00672}
{6, 3.07456}
{7, 3.10996}
Best Fit: (z=delta mu x)
                                      2
```

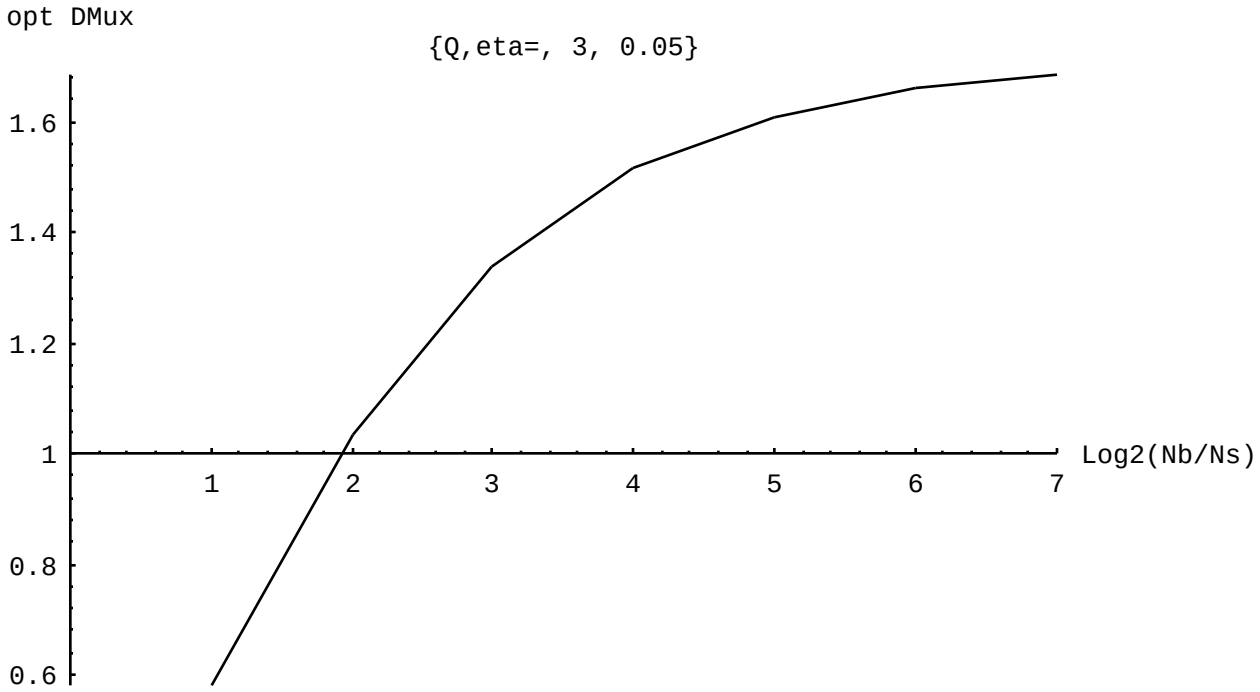
```
1.38431 + 0.548645 z - 0.0437177 z
-------------------------------
Q=6  Eta=0.1
Log2[Nb/Ns]   Optimum Delta Mu x:
{1, 1.71231}
{2, 2.07558}
{3, 2.31417}
{4, 2.45507}
{5, 2.53242}
{6, 2.57307}
{7, 2.59392}
Best Fit: (z=delta mu x)
                                   2
1.36708 + 0.407211 z - 0.0336779 z
-------------------------------
Q=6  Eta=0.15
Log2[Nb/Ns]   Optimum Delta Mu x:
{1, 1.59522}
{2, 1.89915}
{3, 2.0888}
{4, 2.19678}
{5, 2.25475}
{6, 2.28484}
{7, 2.30018}
Best Fit: (z=delta mu x)
                                 2
1.3181 + 0.33194 z - 0.0278666 z
-------------------------------
Q=7  Eta=0
Log2[Nb/Ns]   Optimum Delta Mu x:
{1, 2.30259}
{2, 2.99573}
{3, 3.68888}
{4, 4.38203}
{5, 5.07517}
{6, 5.76832}
{7, 6.46147}
Best Fit: (z=delta mu x)
                                  -8  2
1.60944 + 0.693147 z - 2.38804 10   z
-------------------------------
Q=7  Eta=0.05
Log2[Nb/Ns]   Optimum Delta Mu x:
{1, 2.08516}
{2, 2.55718}
{3, 2.90493}
{4, 3.13123}
{5, 3.26402}
{6, 3.33664}
{7, 3.37473}
Best Fit: (z=delta mu x)
                                   2
1.59069 + 0.562132 z - 0.0444329 z
```

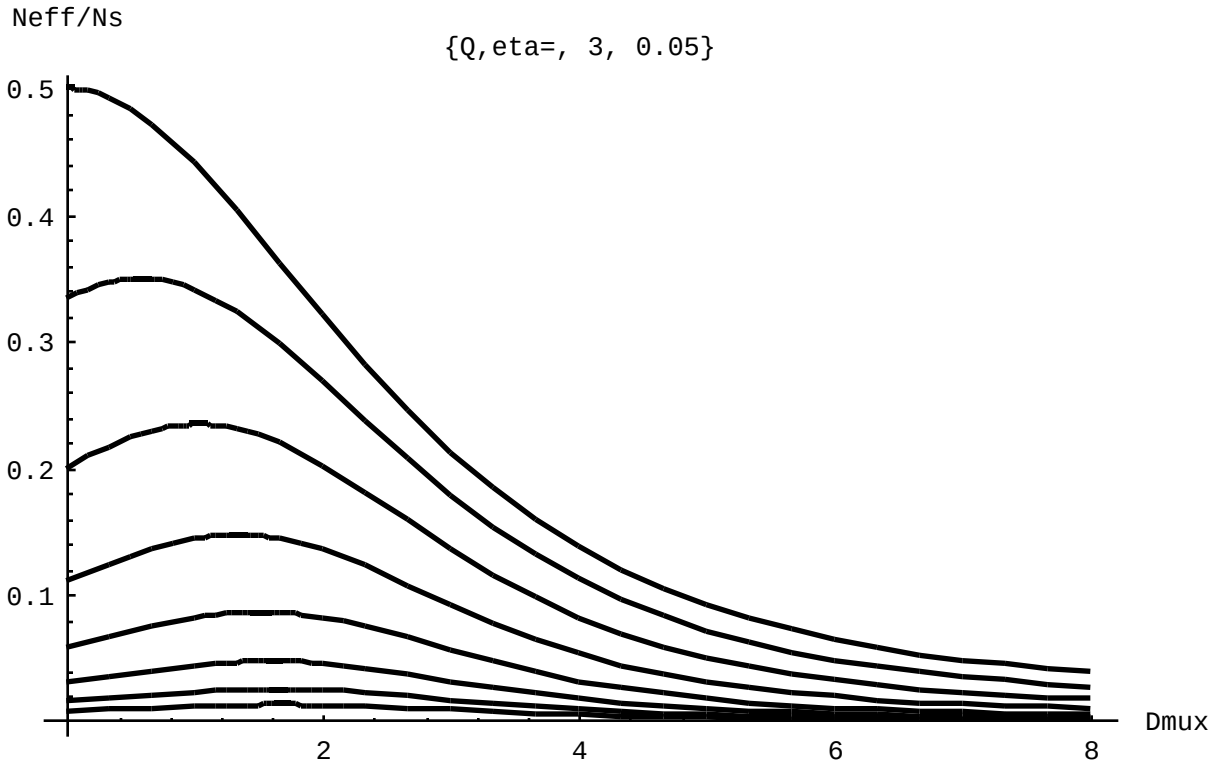
```
-------------------------------
Q=7  Eta=0.1
Log2[Nb/Ns]   Optimum Delta Mu x:
{1, 1.92815}
{2, 2.29835}
{3, 2.54461}
{4, 2.69163}
{5, 2.77293}
{6, 2.81583}
{7, 2.8379}
Best Fit: (z=delta mu x)
                                    2
1.57256 + 0.417725 z - 0.0343918 z
-------------------------------
Q=7  Eta=0.15
Log2[Nb/Ns]   Optimum Delta Mu x:
{1, 1.80573}
{2, 2.11534}
{3, 2.31054}
{4, 2.42258}
{5, 2.48303}
{6, 2.51449}
{7, 2.53055}
Best Fit: (z=delta mu x)
                                   2
1.52105 + 0.339915 z - 0.028448 z
-------------------------------
```

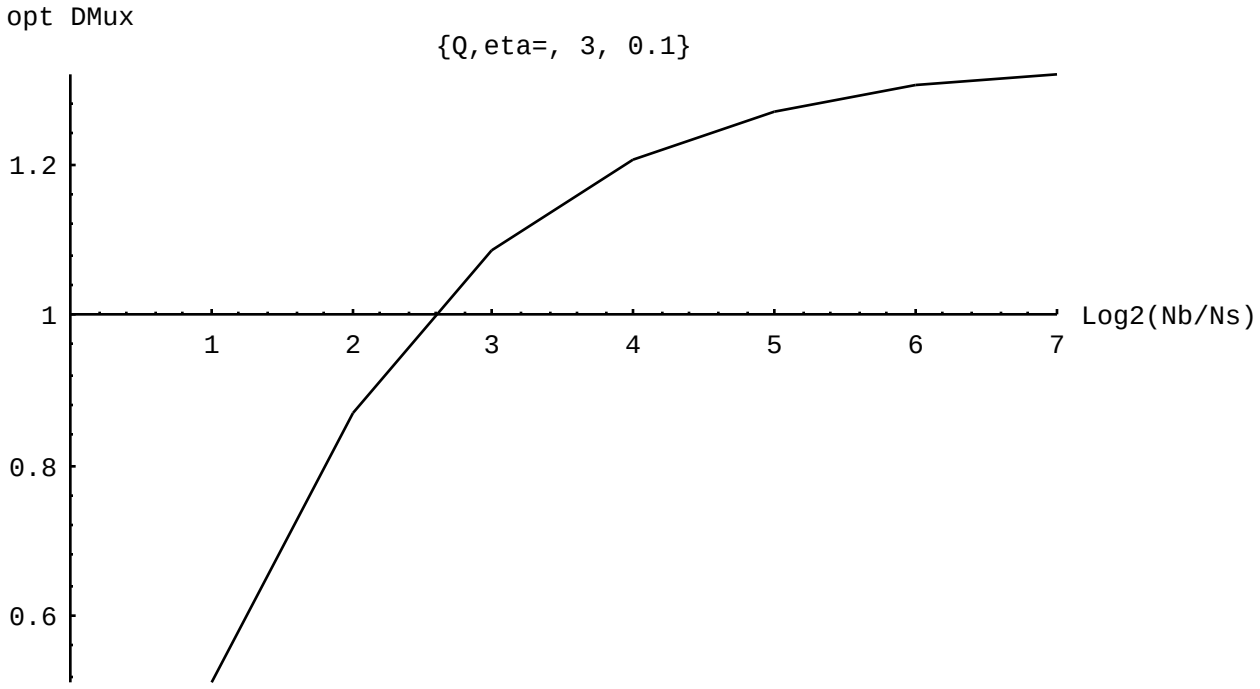
■ Graphs


opt DMux
{Q,eta=, 3, 0}
4
3
2
1
1
2
3
4
5
6
7
Log2(Nb/Ns)


Neff/Ns
{Q,eta=, 3, 0}
0.5
0.4
0.3
0.2
0.1
2
4
6
8
Dmux


opt DMux
{Q,eta=, 3, 0.05}
1.6
1.4
1.2
1
0.8
0.6
1
2
3
4
5
6
7
Log2(Nb/Ns)


Neff/Ns
{Q,eta=, 3, 0.05}
0.5
0.4
0.3
0.2
0.1
2
4
6
8
Dmux


opt DMux
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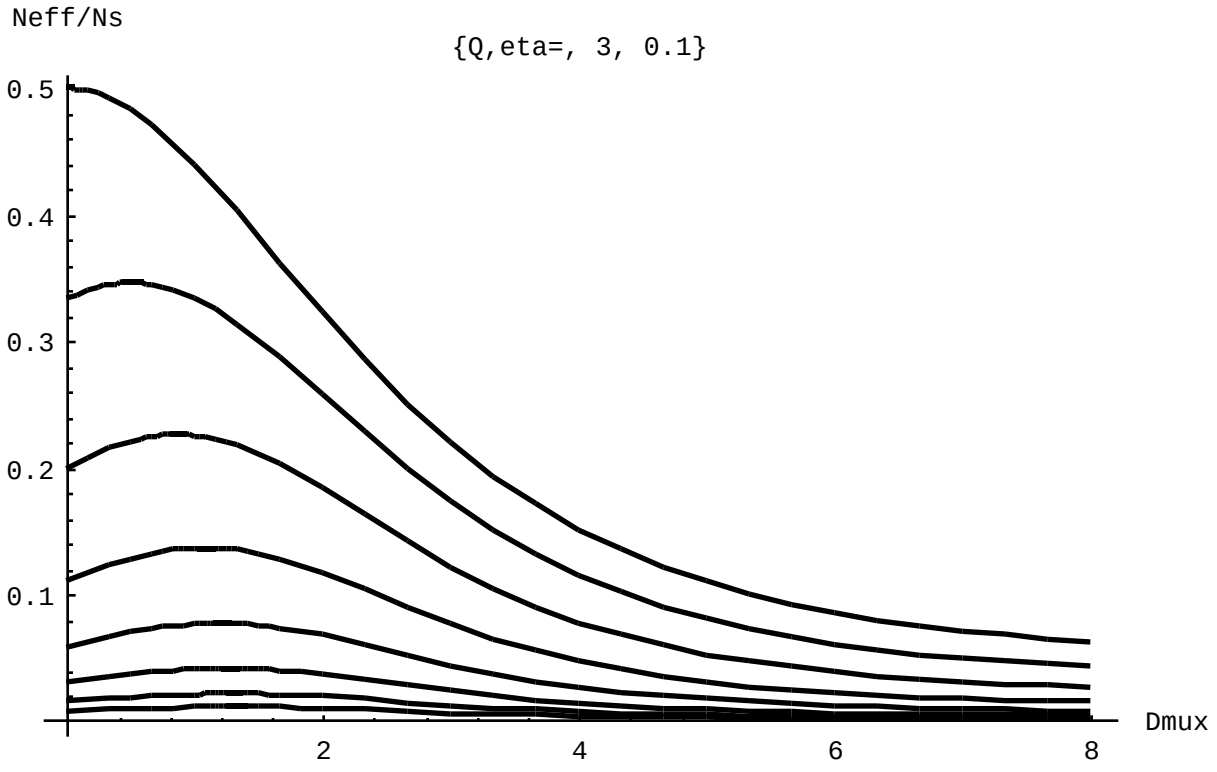
1.2
1
0.8
0.6
1
2
3
4
5
6
7
Log2(Nb/Ns)


Neff/Ns
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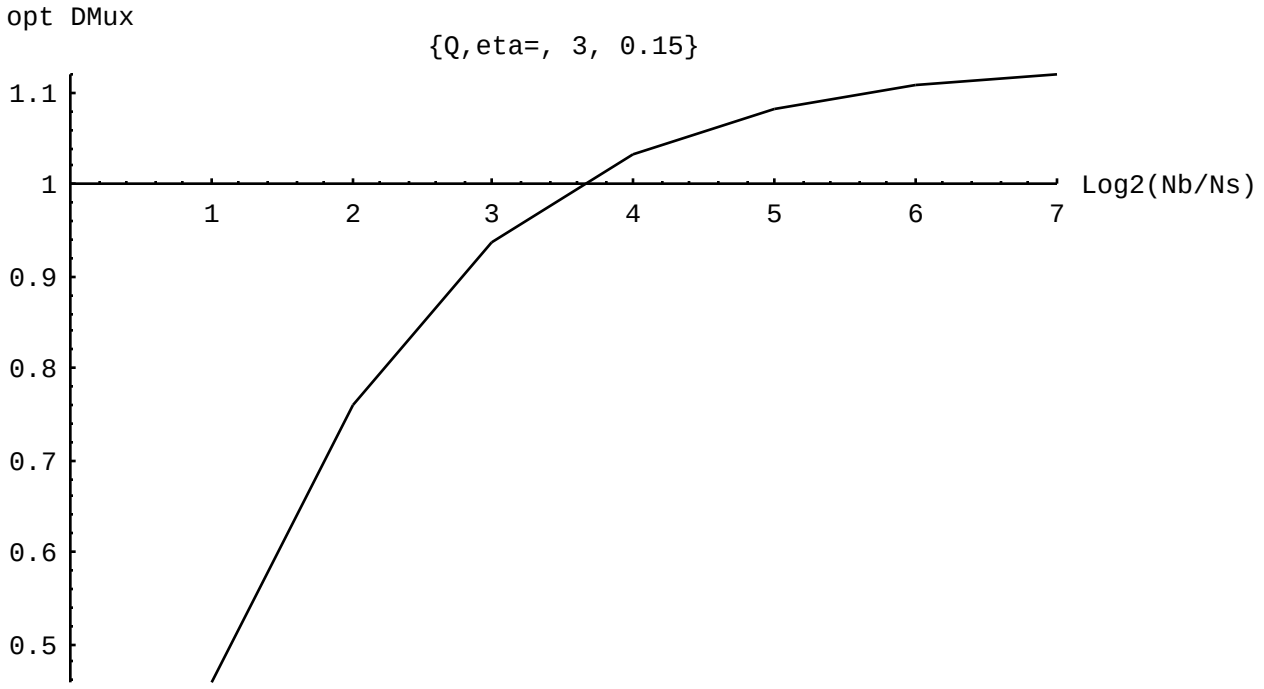
0.5
0.4
0.3
0.2
0.1
2
4
6
8
Dmux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1
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0.7
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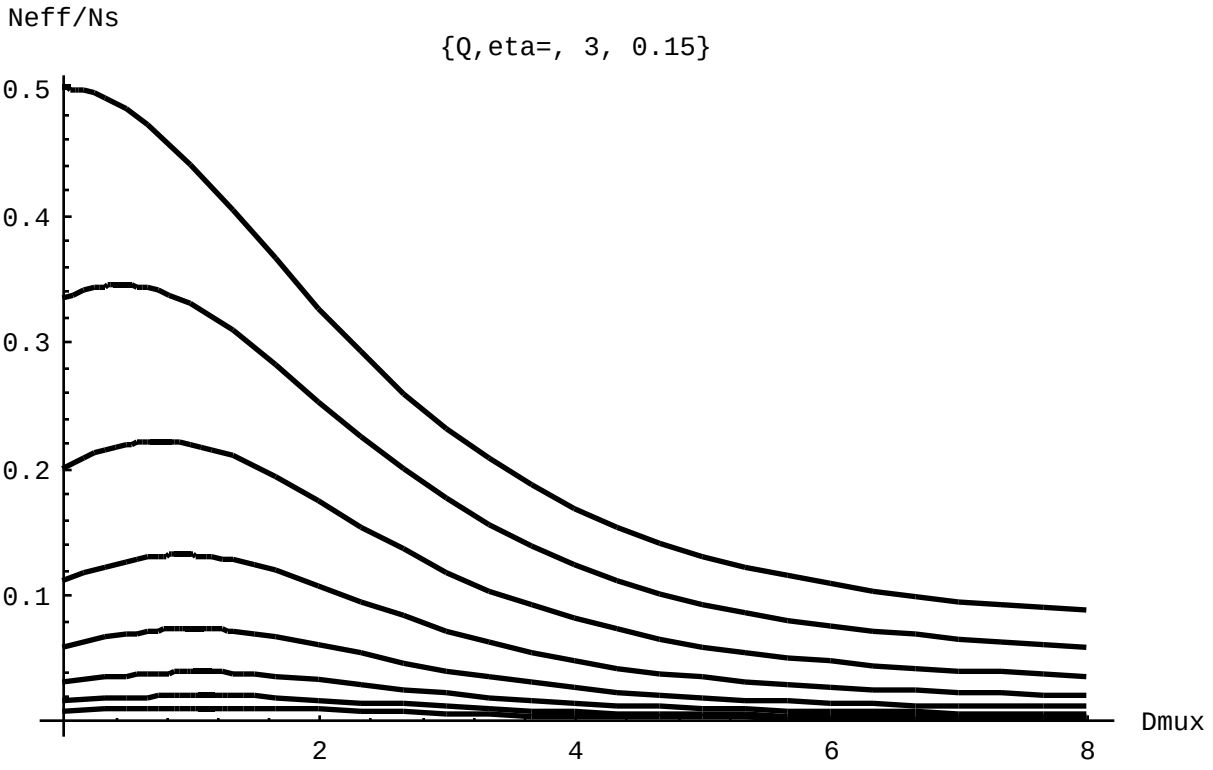
0.5
1
2
3
4
5
6
7
Log2(Nb/Ns)


Neff/Ns
{Q,eta=, 3, 0.15}
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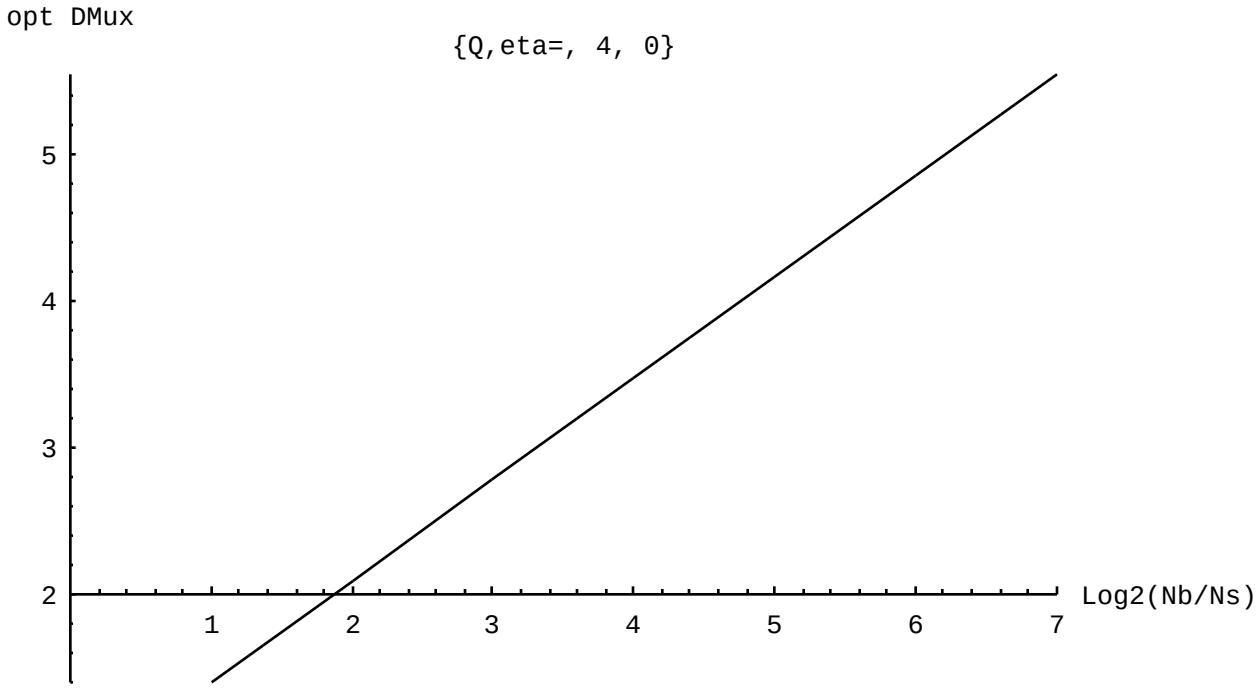
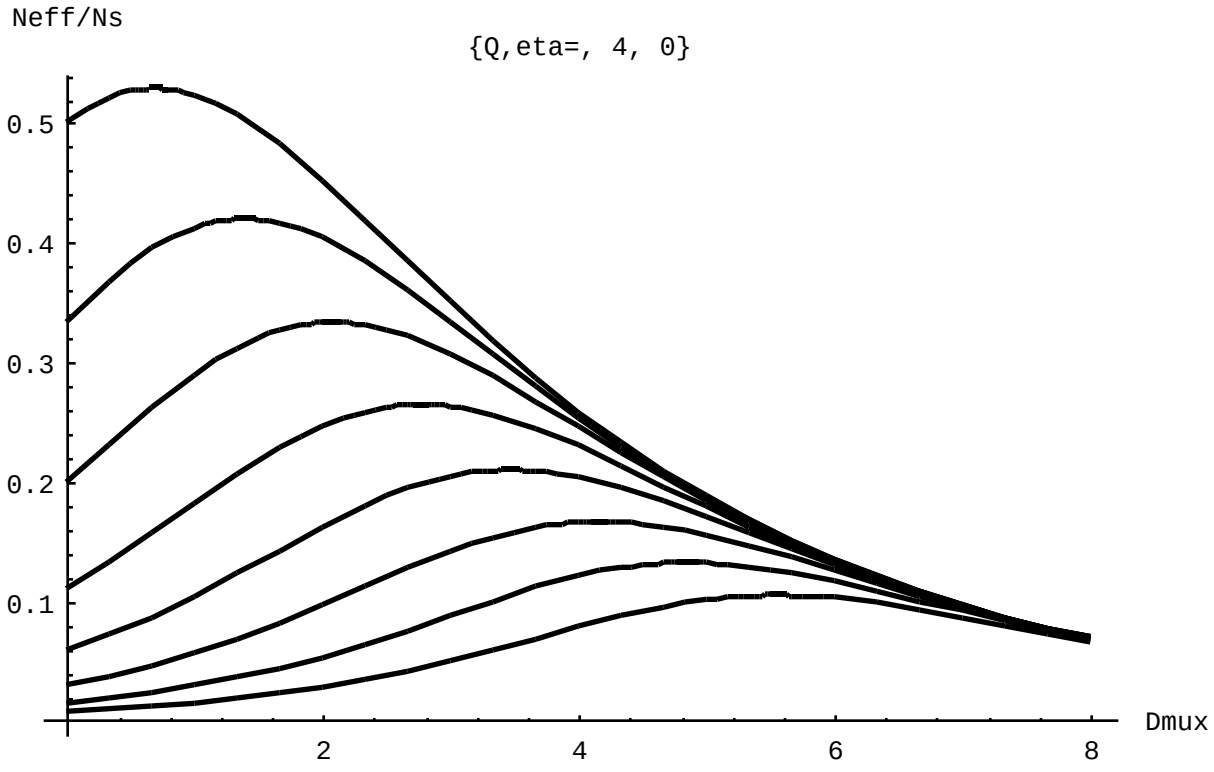
0.4
0.3
0.2
0.1
2
4
6
8
Dmux


opt DMux
{Q,eta=, 4, 0}
5
4
3
2
1
2
3
4
5
6
7
Log2(Nb/Ns)


Neff/Ns
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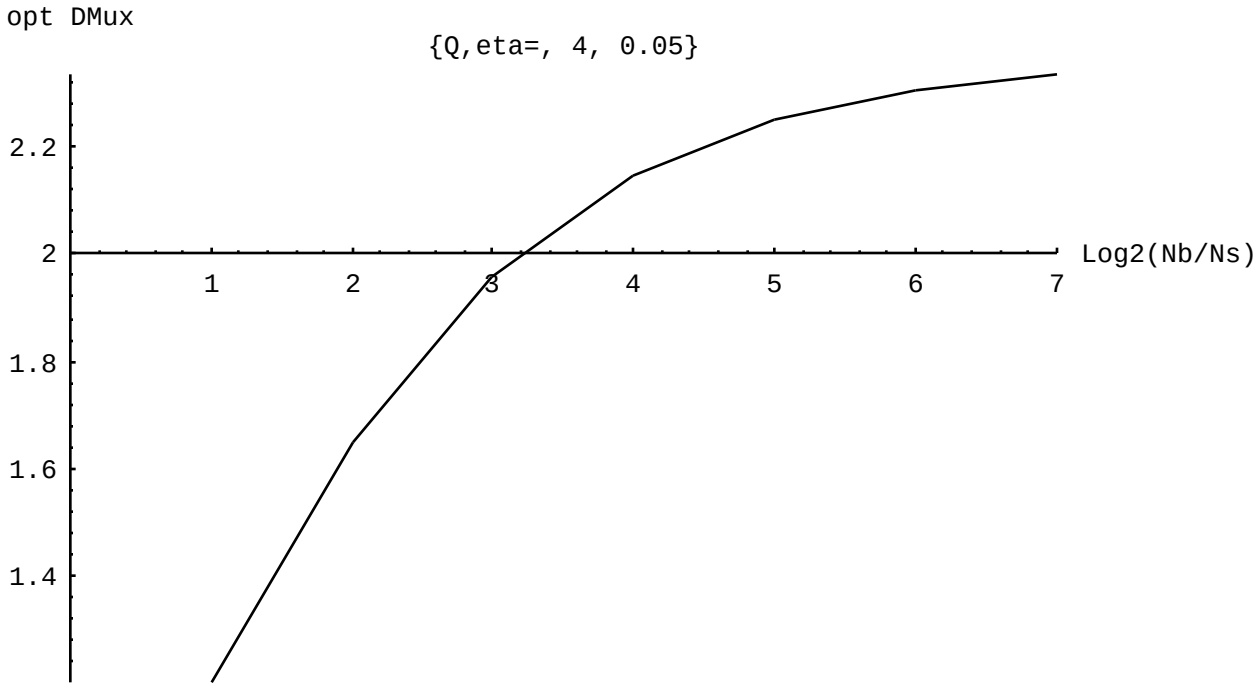
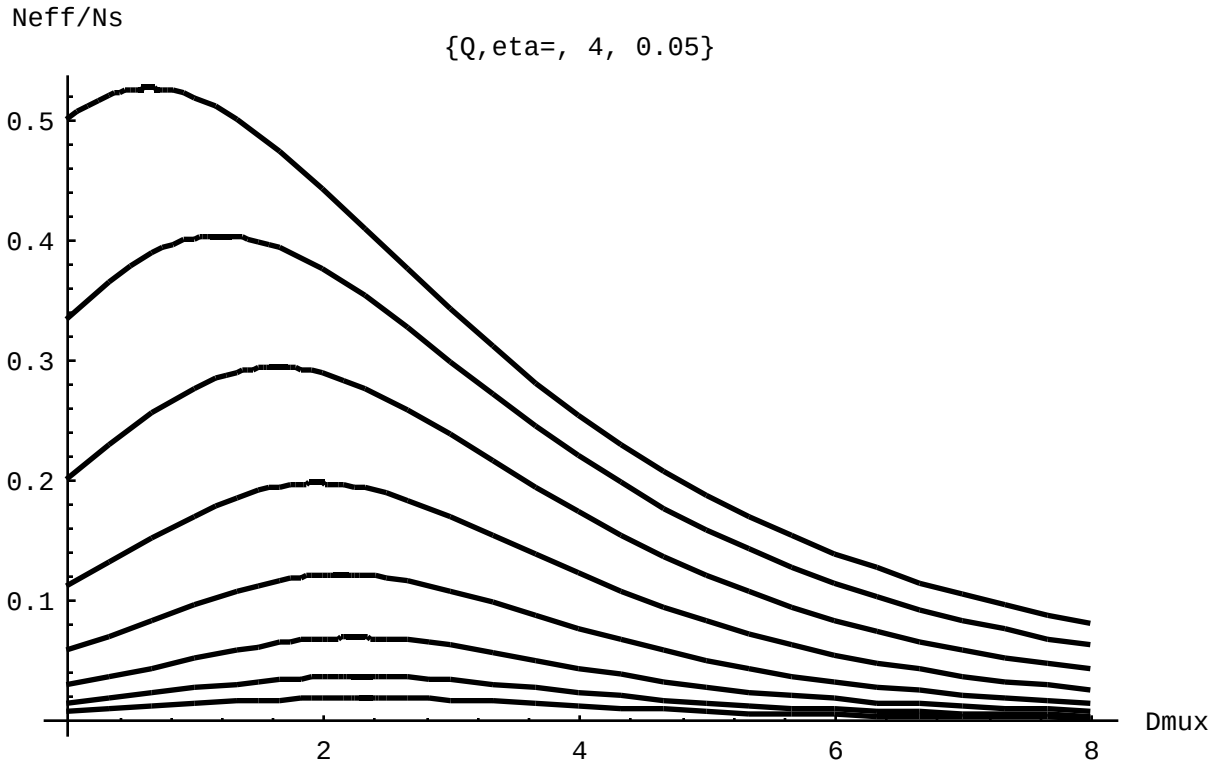
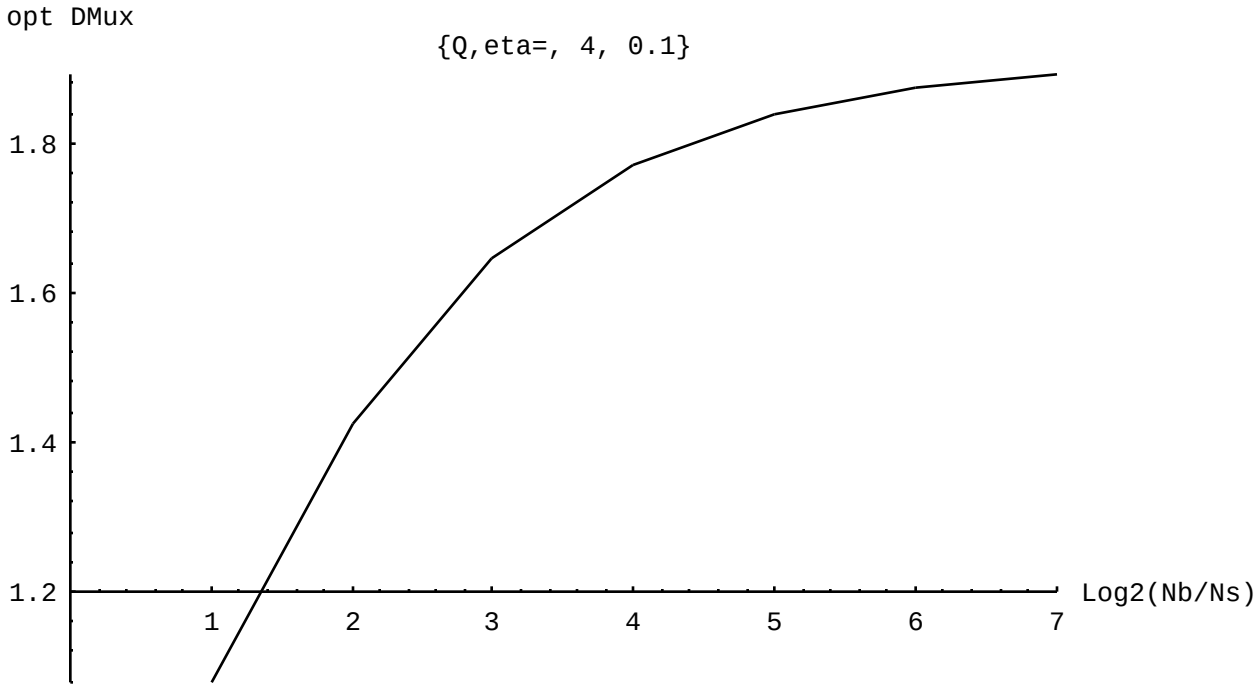
0.5
0.4
0.3
0.2
0.1
2
4
6
8
Dmux


opt DMux
{Q,eta=, 4, 0.05}
2.2
2
1.8
1.6
1.4
1
2
3
4
5
6
7
Log2(Nb/Ns)


Neff/Ns
{Q,eta=, 4, 0.05}
0.5
0.4
0.3
0.2
0.1
2
4
6
8
Dmux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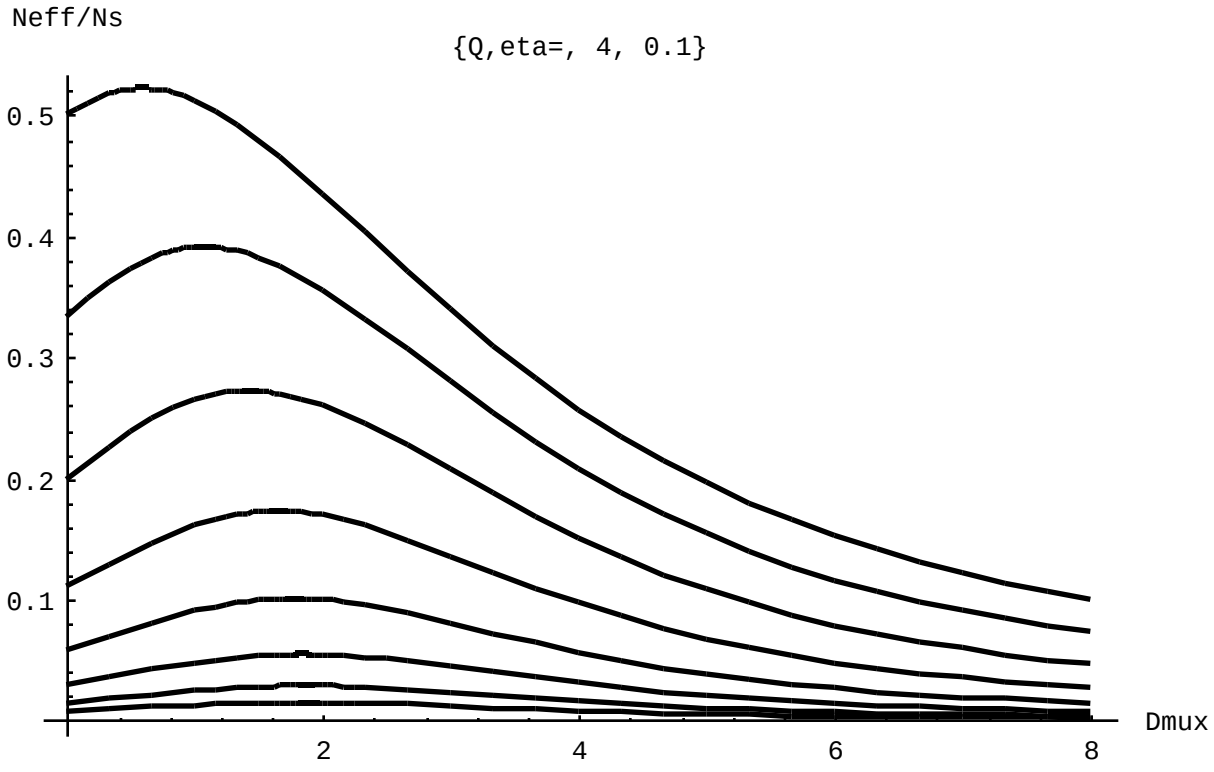
1.6
1.4
1.2
1
2
3
4
5
6
7
Log2(Nb/Ns)


Neff/Ns
{Q,eta=, 4, 0.1}
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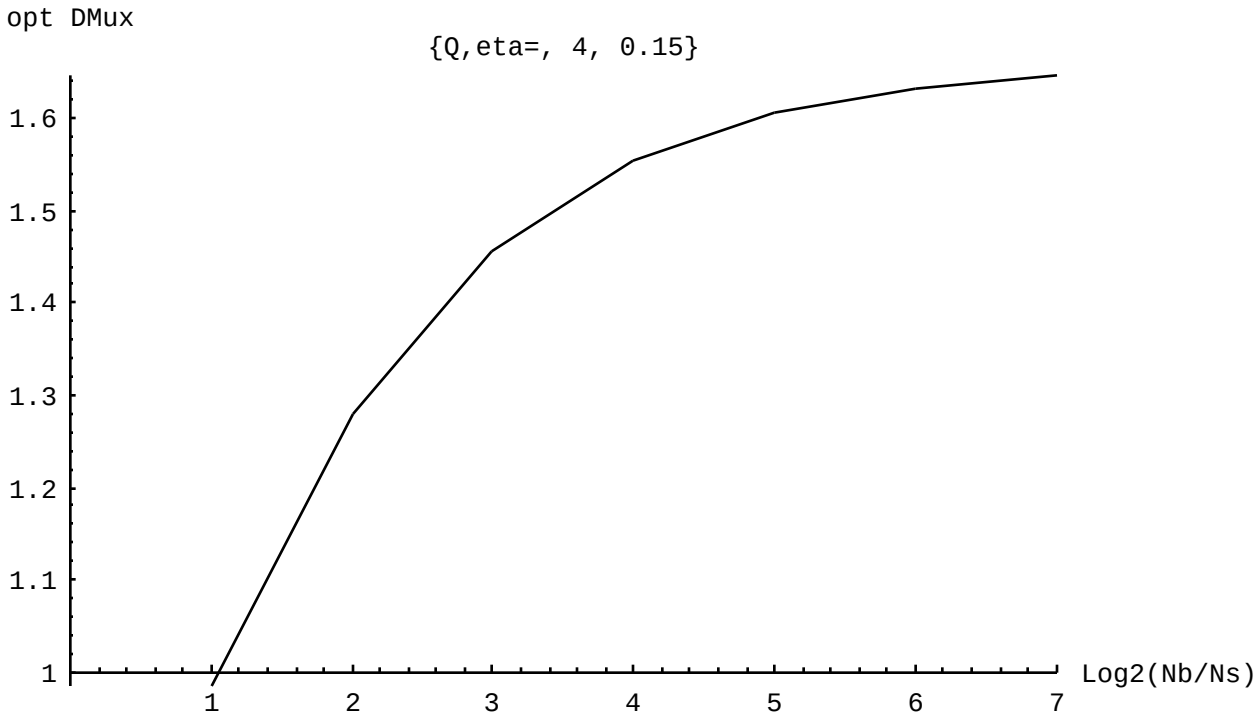
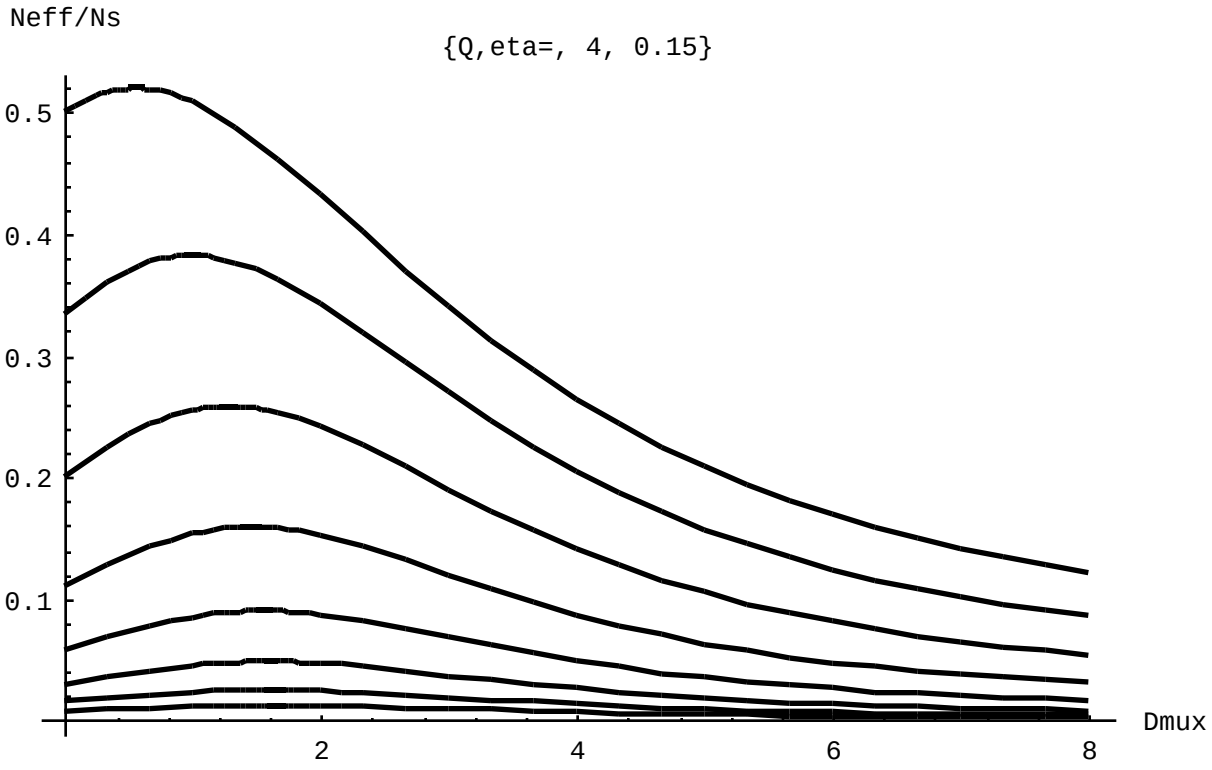
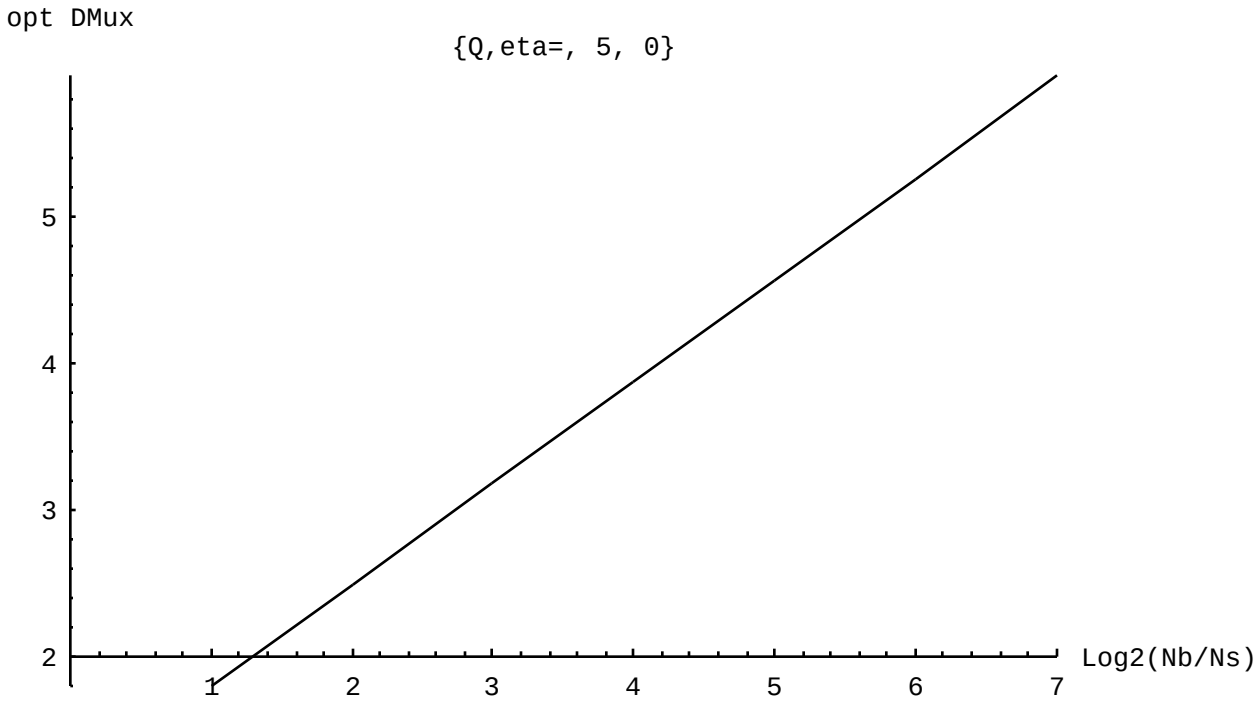
0.4
0.3
0.2
0.1
2
4
6
8
Dmux


opt DMux
{Q,eta=, 4, 0.15}
1.6
1.5
1.4
1.3
1.2
1.1
1
1
2
3
4
5
6
7
Log2(Nb/Ns)


Neff/Ns
{Q,eta=, 4, 0.15}
0.5
0.4
0.3
0.2
0.1
2
4
6
8
Dmux


opt DMux
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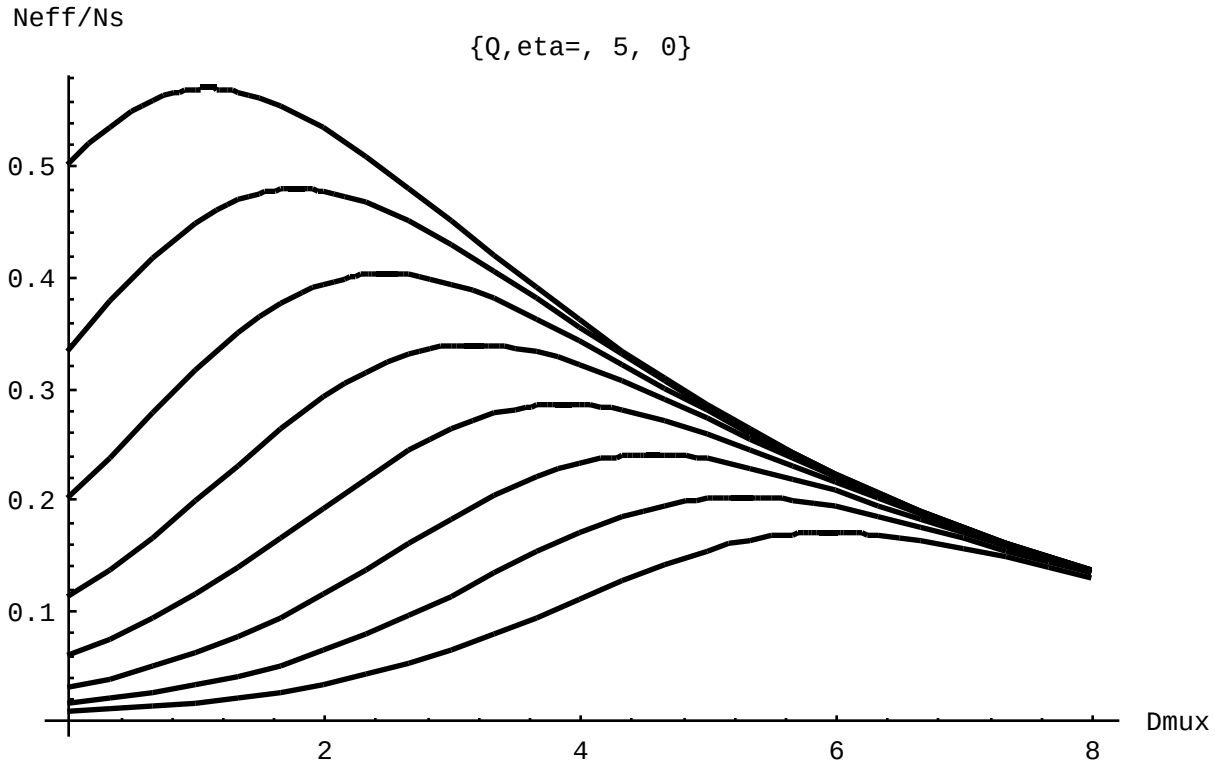
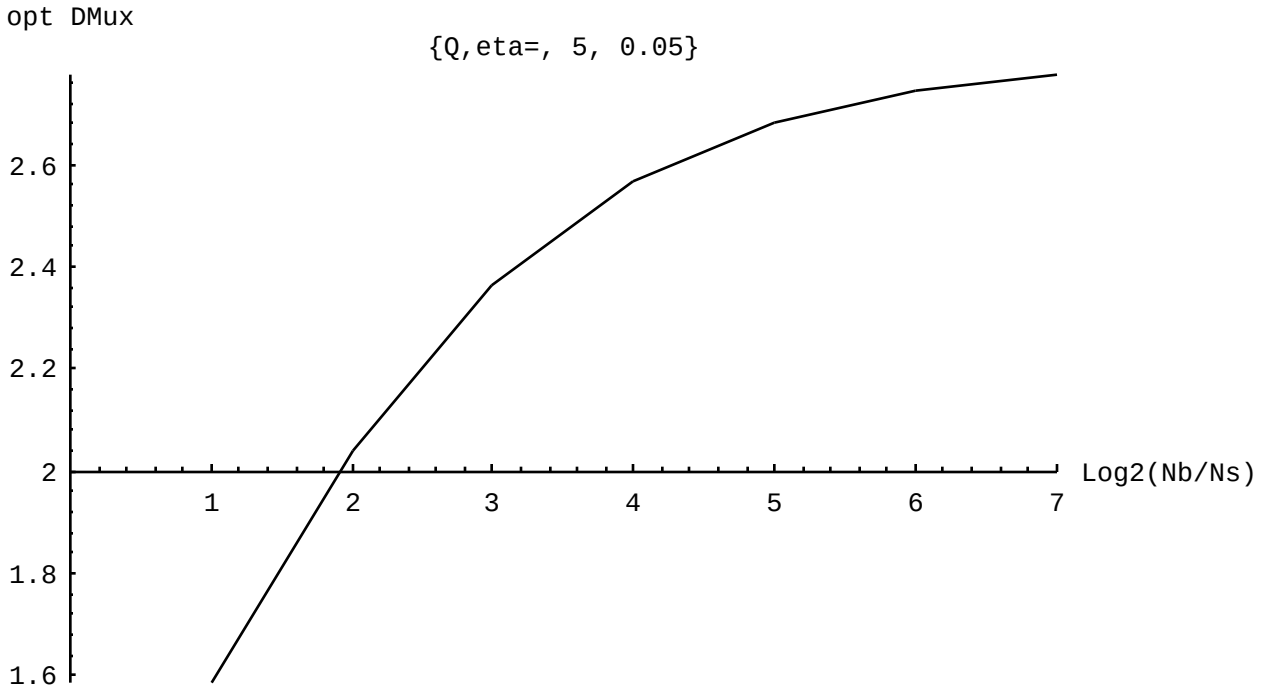
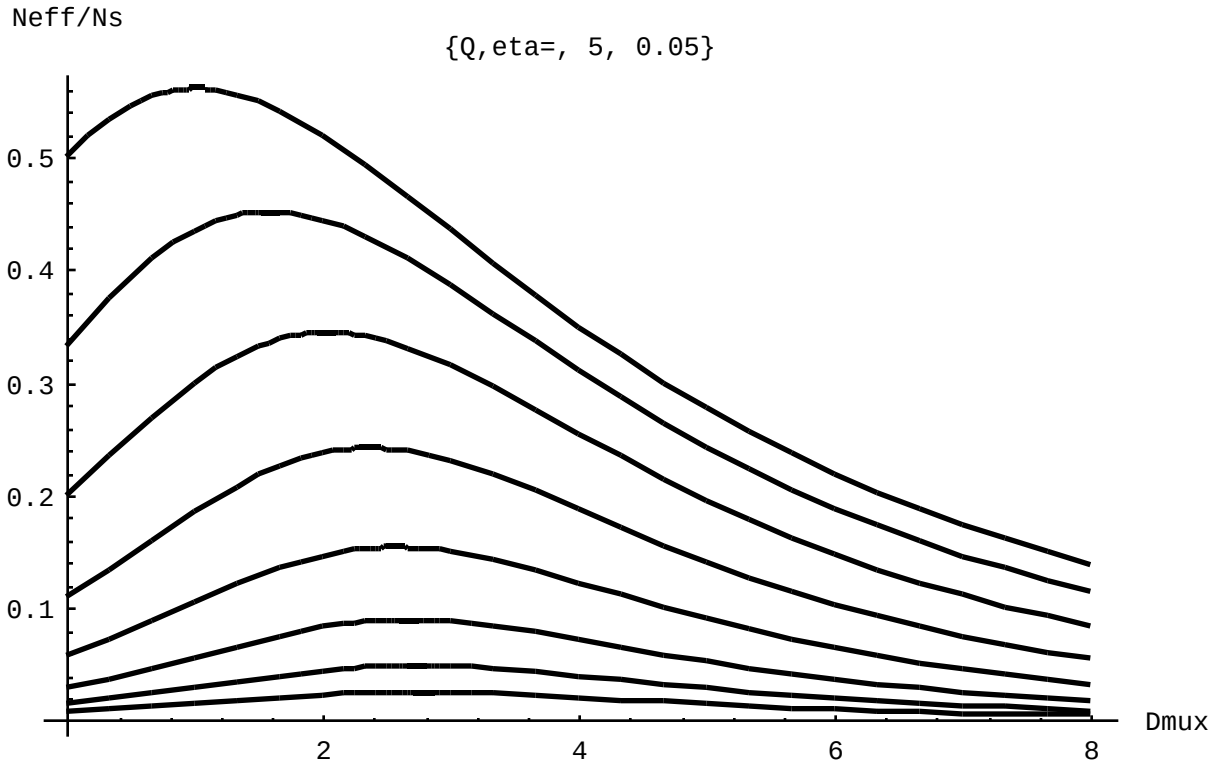
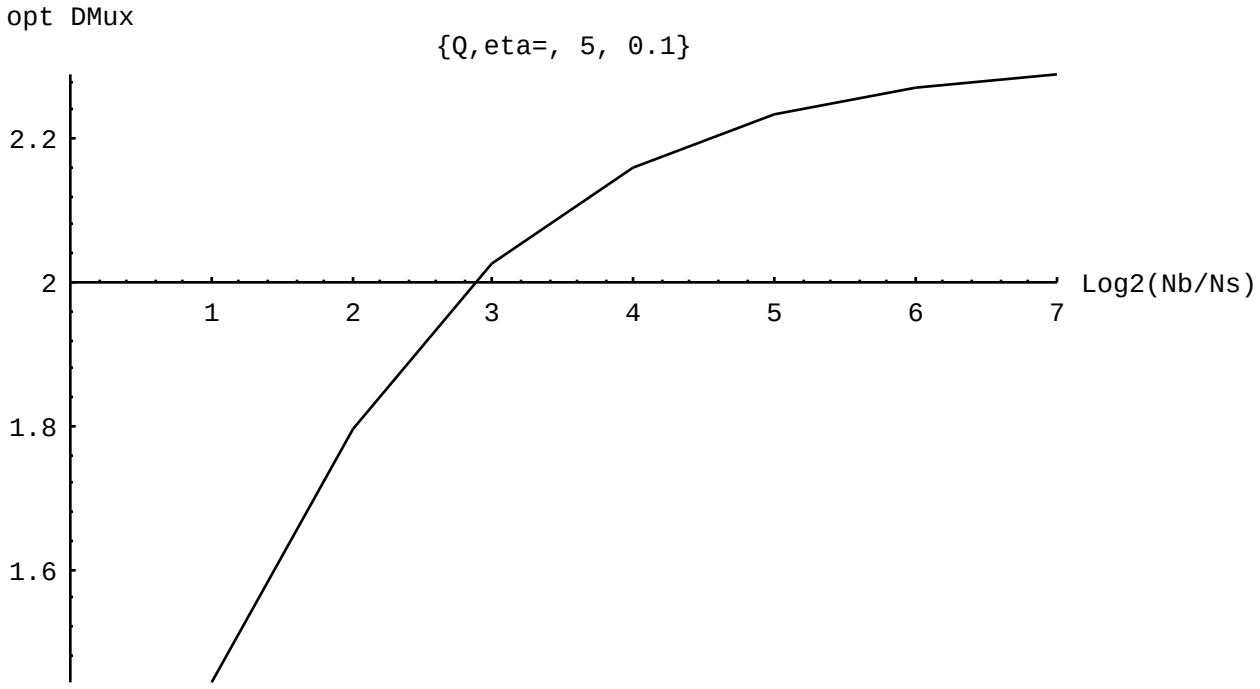
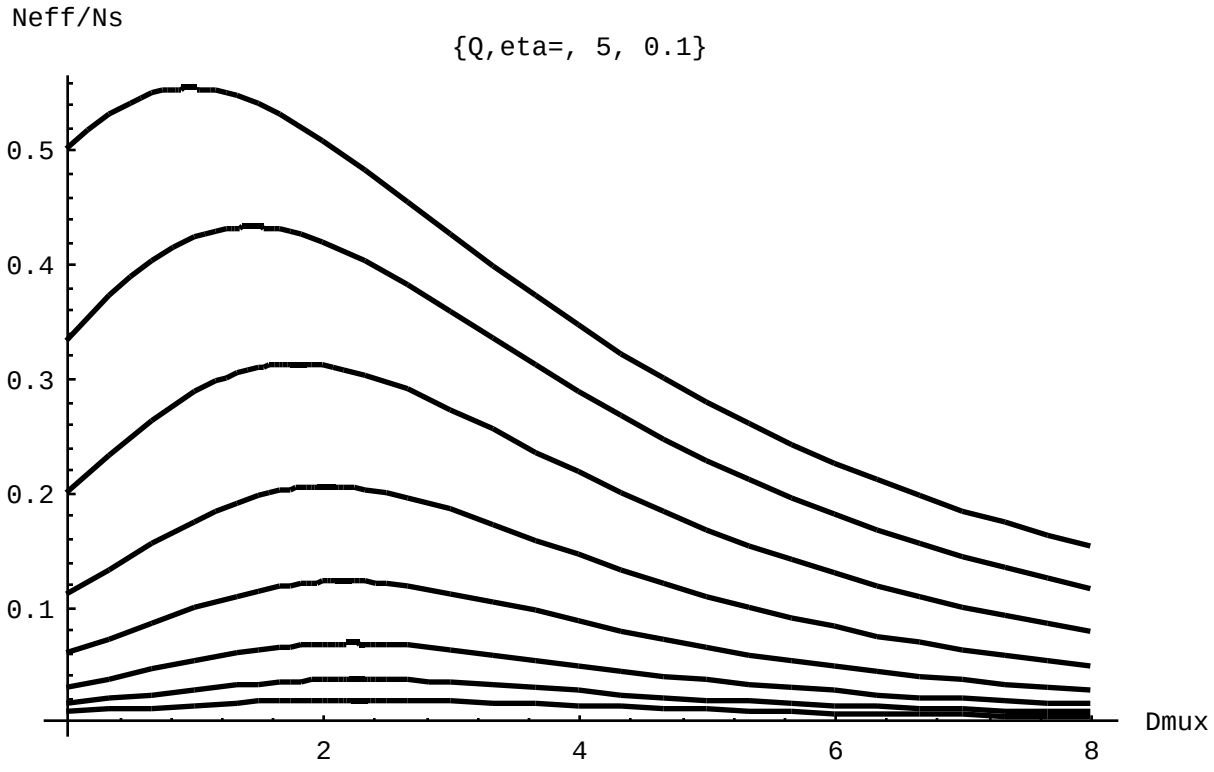
5
4
3
2
1
2
3
4
5
6
7
Log2(Nb/Ns)

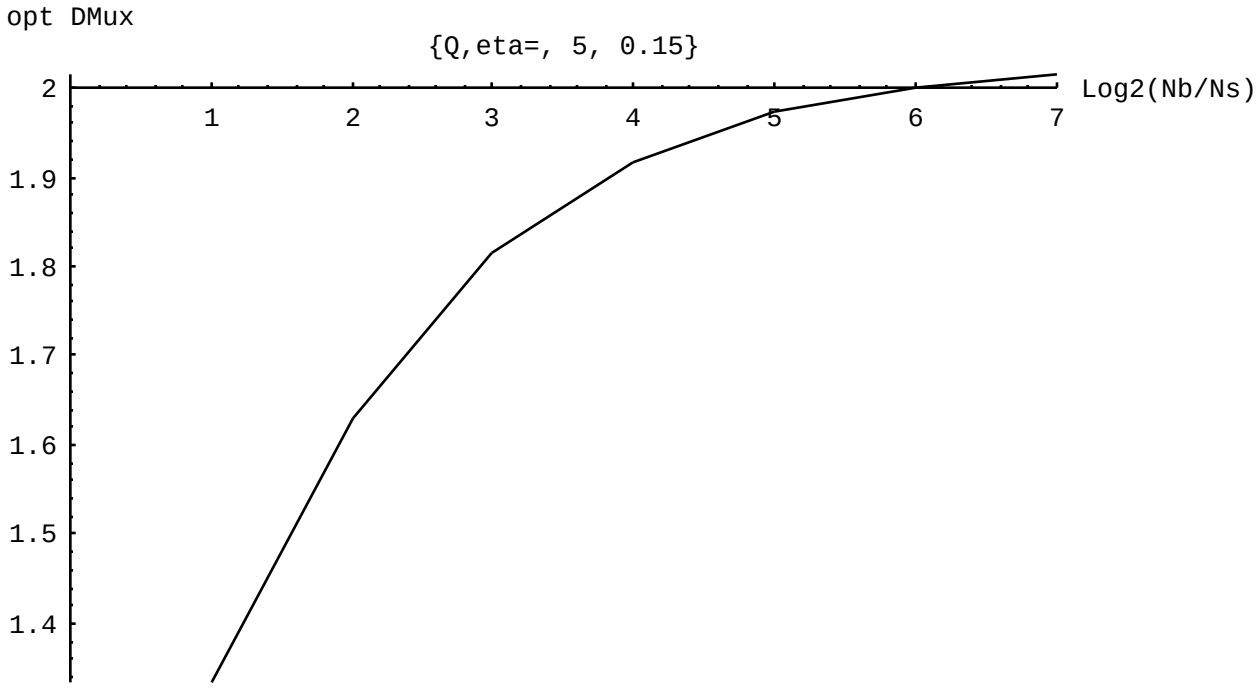

Neff/Ns
{Q,eta=, 5, 0}
0.5
0.4
0.3
0.2
0.1
2
4
6
8
Dmux

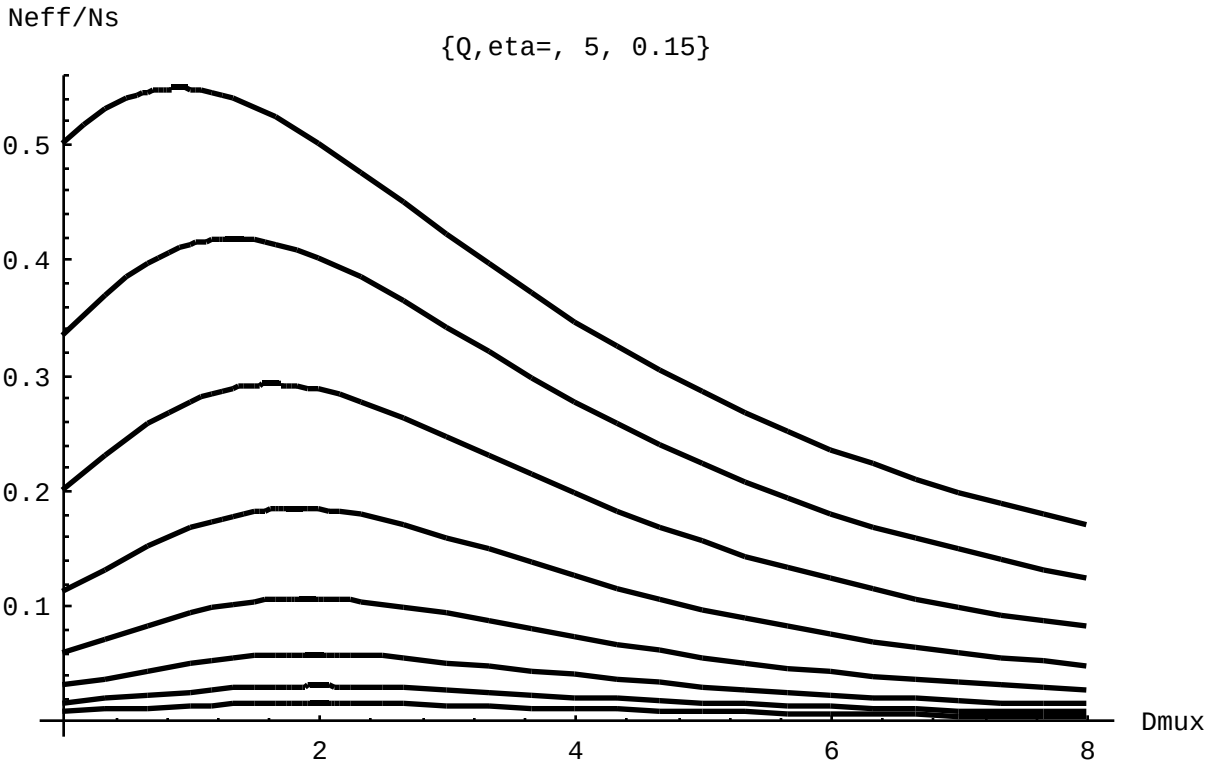

opt DMux
{Q,eta=, 5, 0.05}
2.6
2.4
2.2
2
1.8
1.6
1
2
3
4
5
6
7
Log2(Nb/Ns)

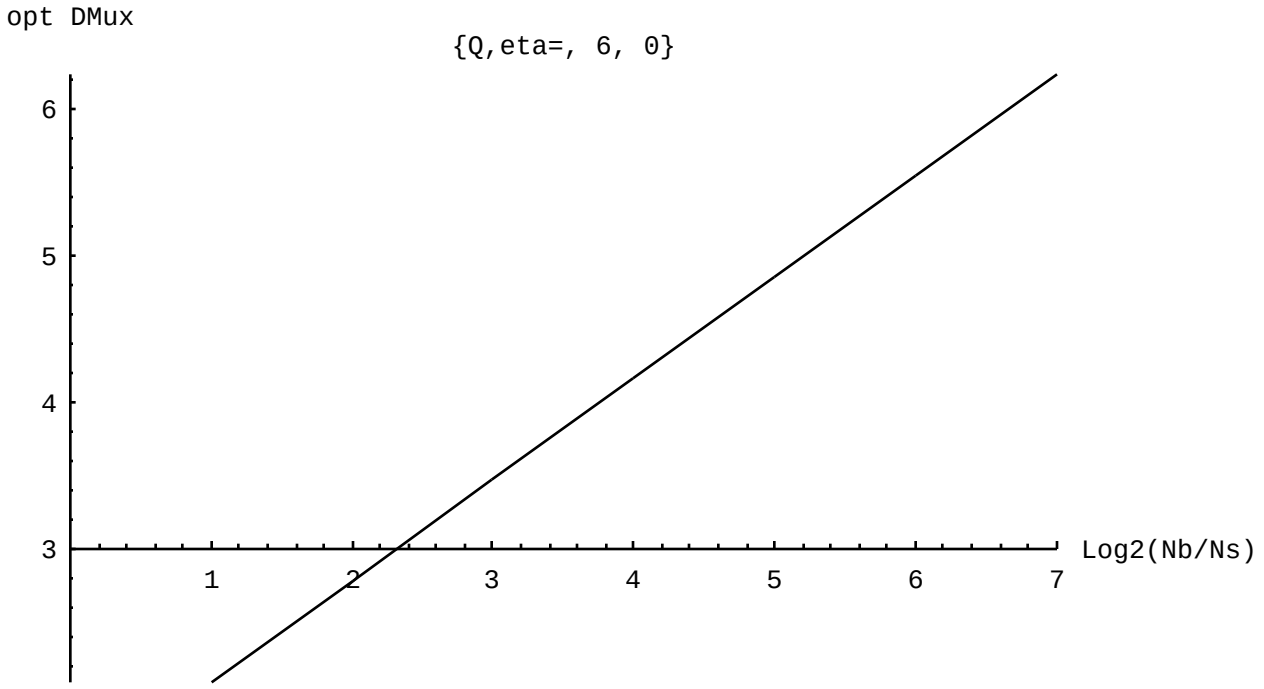

Neff/Ns
{Q,eta=, 5, 0.05}
0.5
0.4
0.3
0.2
0.1
2
4
6
8
Dmux

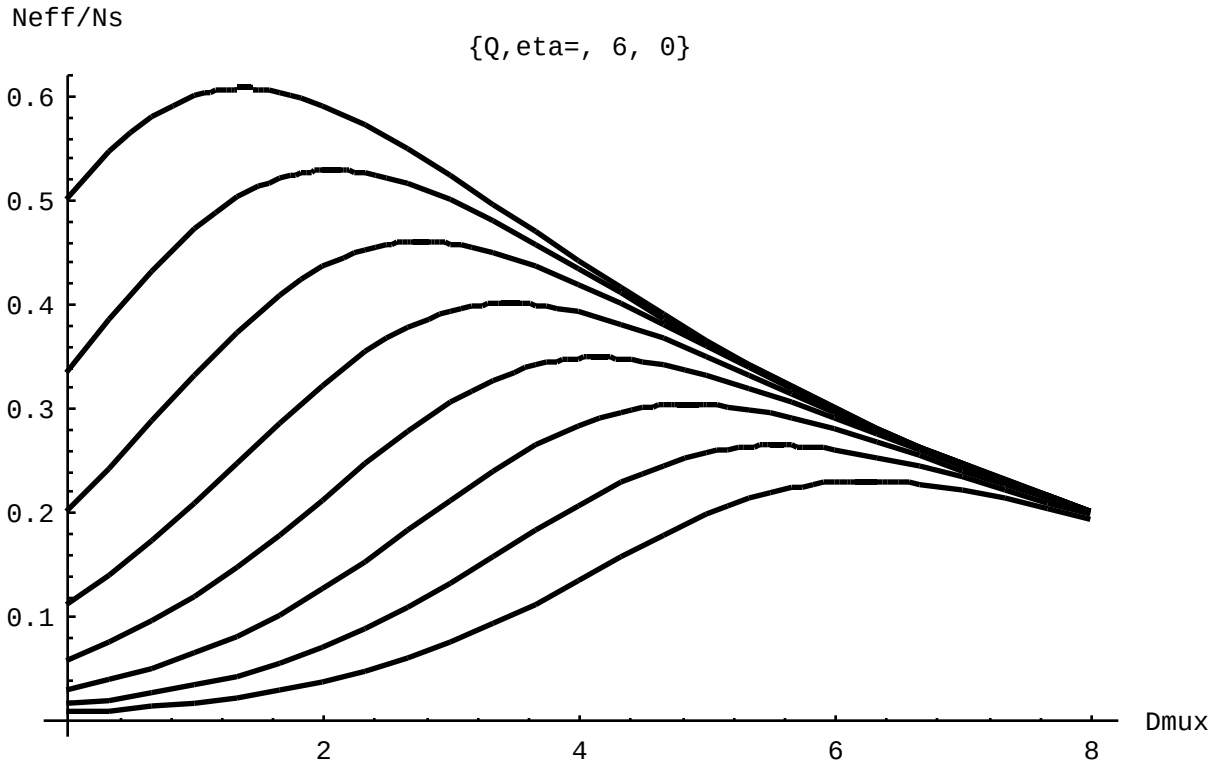

opt DMux
{Q,eta=, 5, 0.1}
2.2
2
1.8
1.6
1
2
3
4
5
6
7
Log2(Nb/Ns)

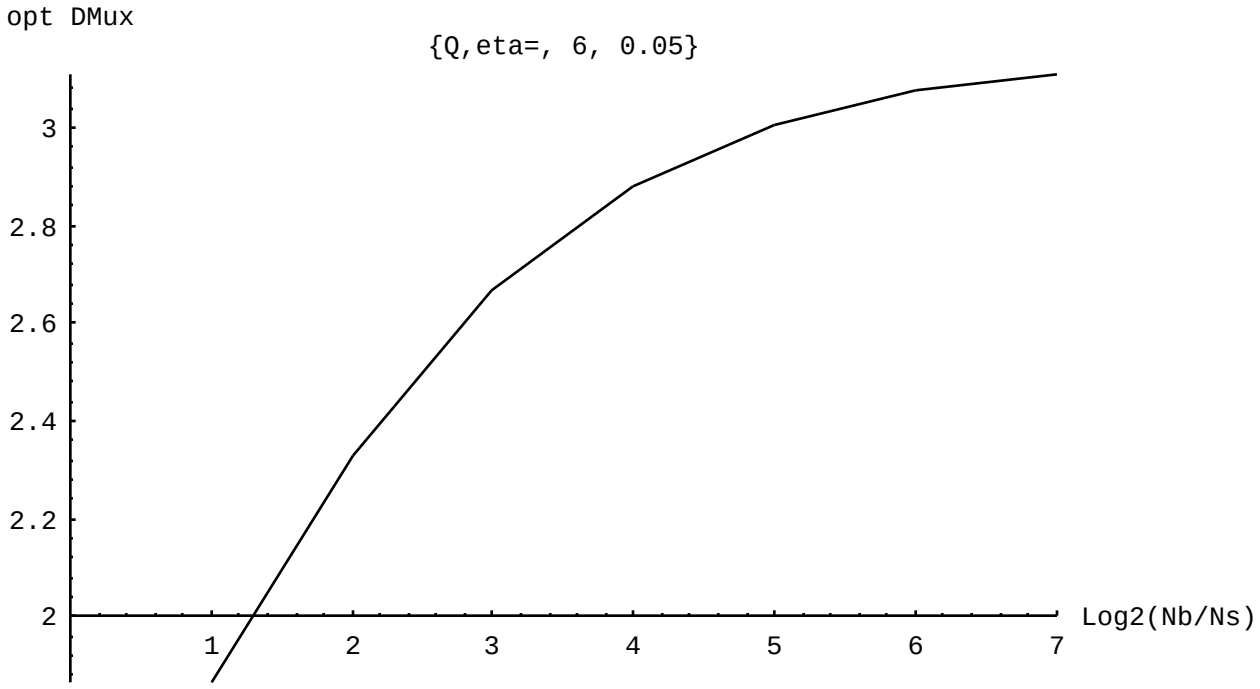

Neff/Ns
{Q,eta=, 5, 0.1}
0.5
0.4
0.3
0.2
0.1
2
4
6
8
Dmux


opt DMux
{Q,eta=, 5, 0.15}
Log2(Nb/Ns)
2
1.9
1.8
1.7
1.6
1.5
1.4
1
2
3
4
5
6
7


Neff/Ns
{Q,eta=, 5, 0.15}
Dmux
0.5
0.4
0.3
0.2
0.1
2
4
6
8


opt DMux
{Q,eta=, 6, 0}
6
5
4
3
1
2
3
4
5
6
7
Log2(Nb/Ns)


Neff/Ns
{Q,eta=, 6, 0}
0.6
0.5
0.4
0.3
0.2
0.1
2
4
6
8
Dmux


opt DMux
{Q,eta=, 6, 0.05}
3
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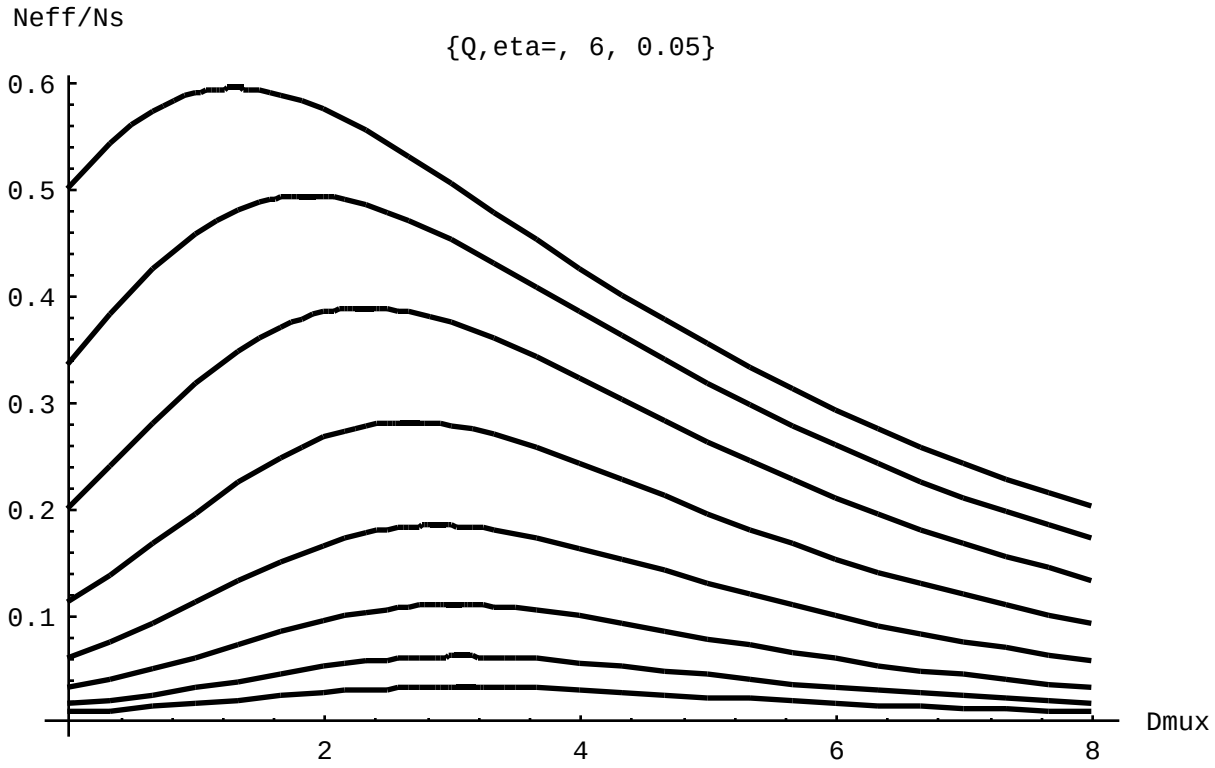
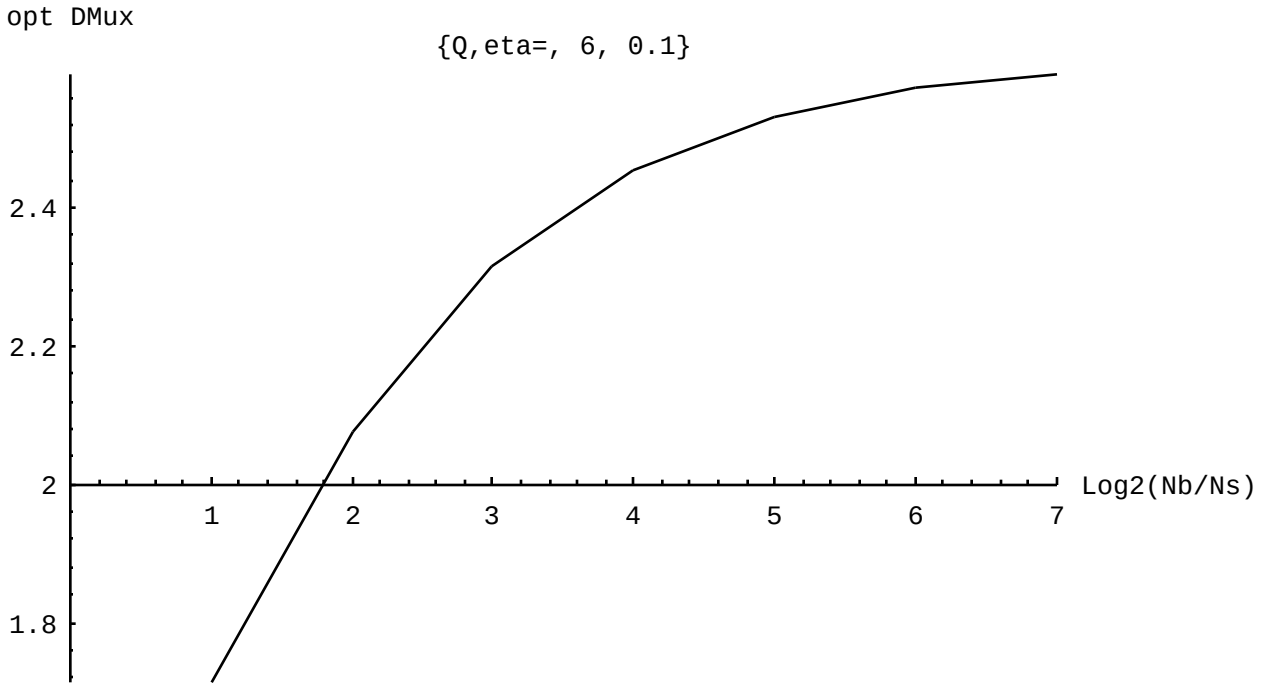
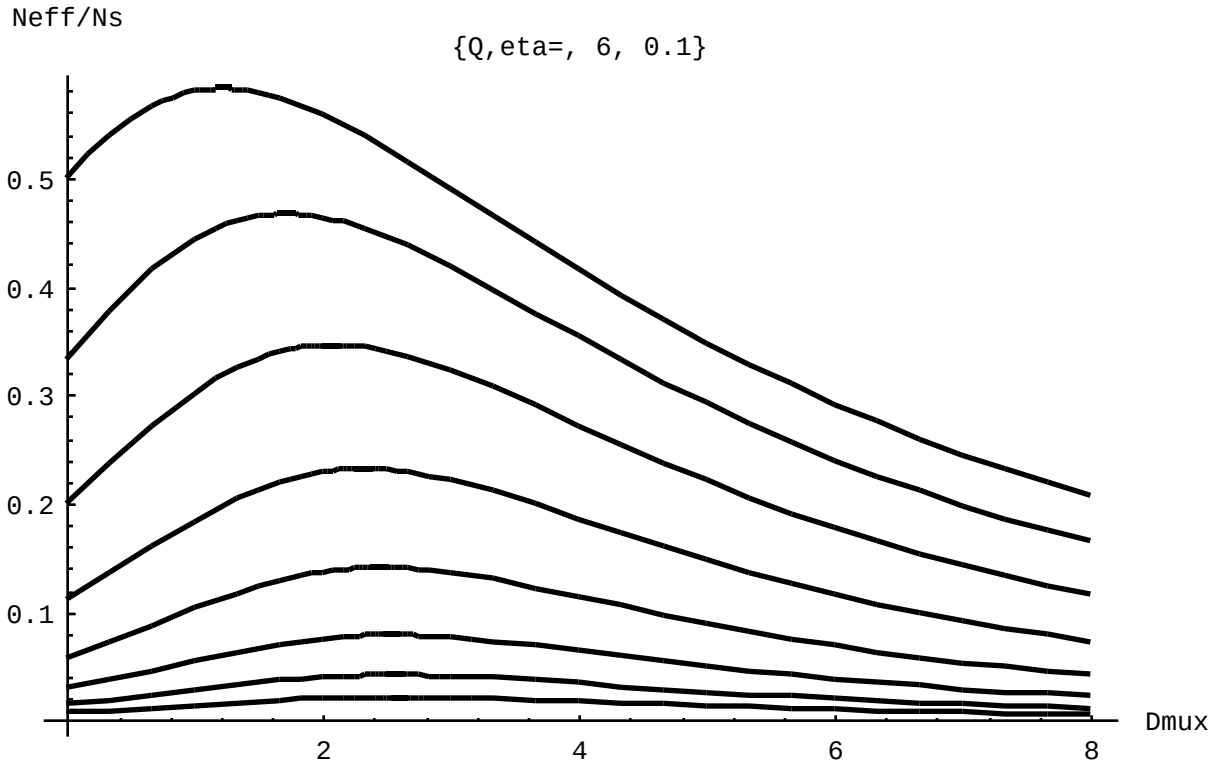
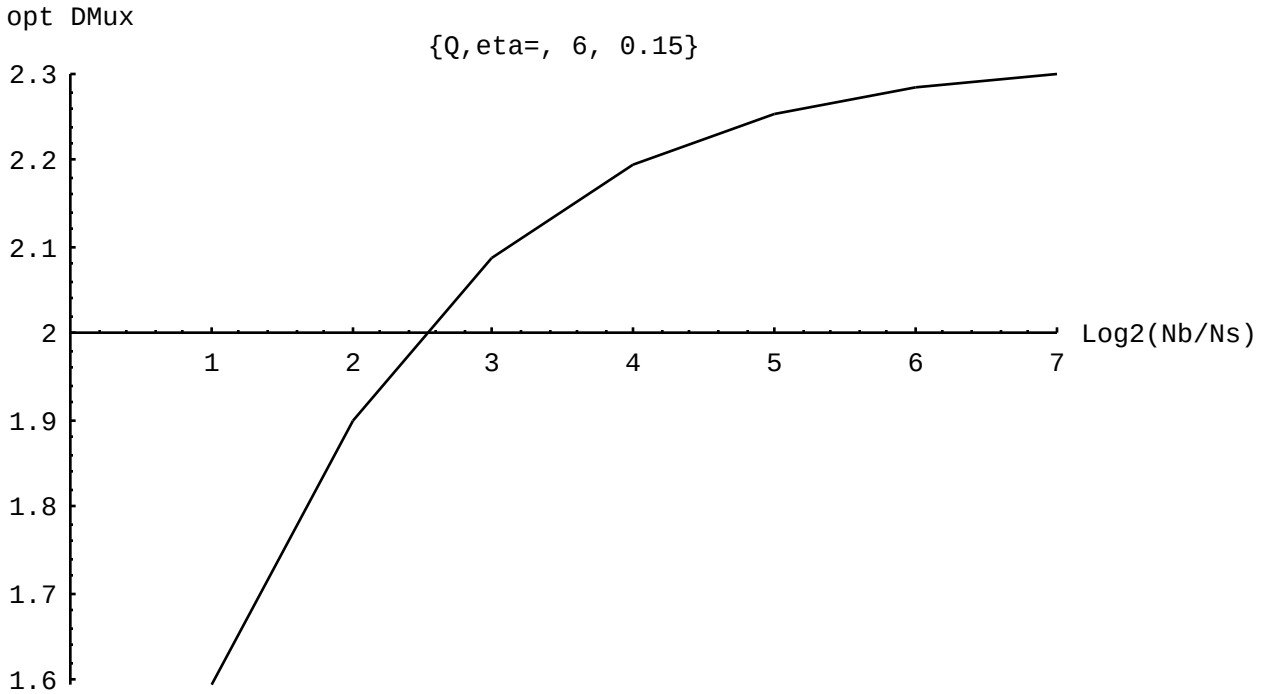
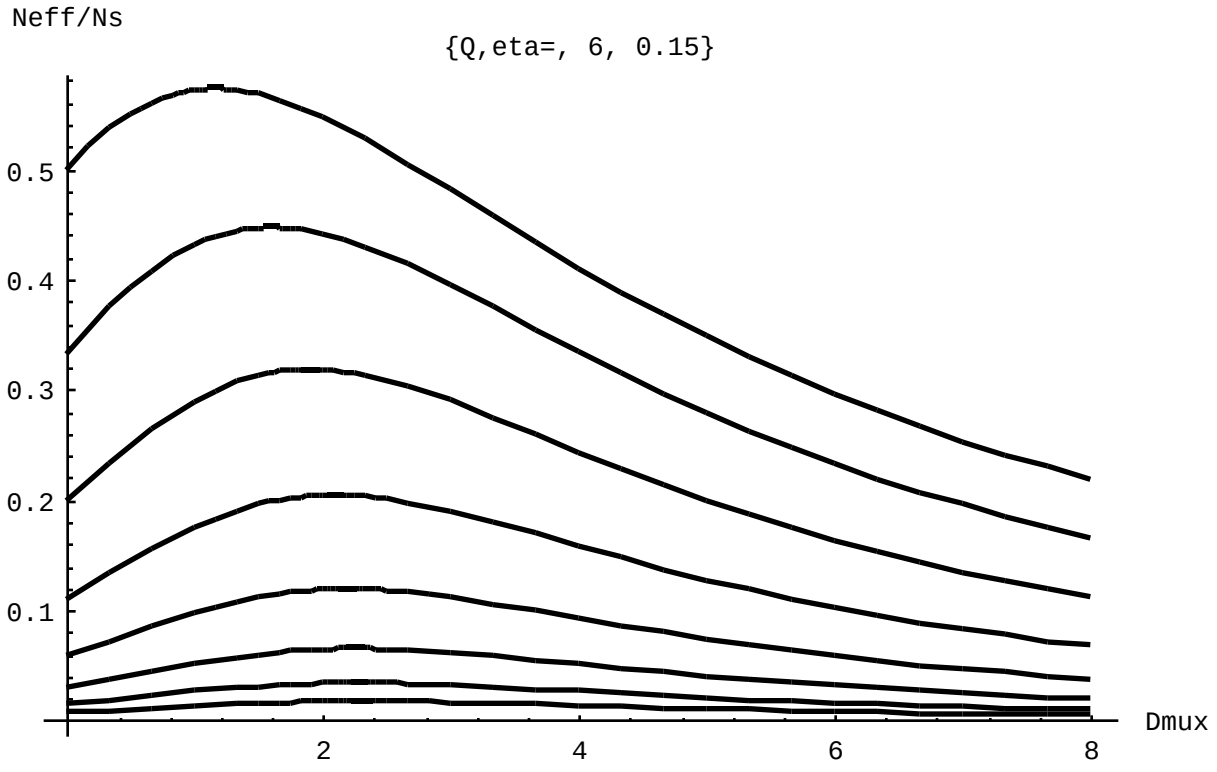
2.6
2.4
2.2
2
1
2
3
4
5
6
7
Log2(Nb/Ns)

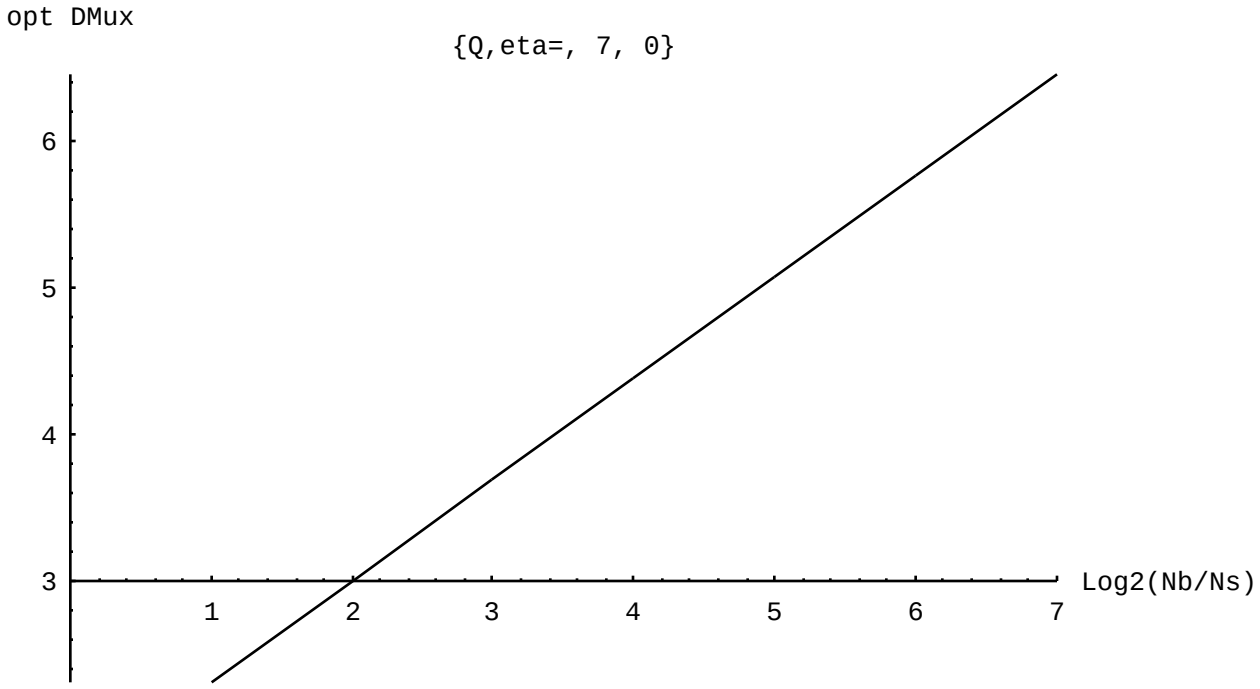

Neff/Ns
{Q,eta=, 6, 0.05}
0.6
0.5
0.4
0.3
0.2
0.1
2
4
6
8
Dmux


opt DMux
{Q,eta=, 6, 0.1}
2.4
2.2
2
1.8
1
2
3
4
5
6
7
Log2(Nb/Ns)


Neff/Ns
{Q,eta=, 6, 0.1}
0.5
0.4
0.3
0.2
0.1
2
4
6
8
Dmux


opt DMux
{Q,eta=, 6, 0.15}
2.3
2.2
2.1
2
1.9
1.8
1.7
1.6
1
2
3
4
5
6
7
Log2(Nb/Ns)


Neff/Ns
{Q,eta=, 6, 0.15}
0.5
0.4
0.3
0.2
0.1
2
4
6
8
Dmux


opt DMux
{Q,eta=, 7, 0}
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5
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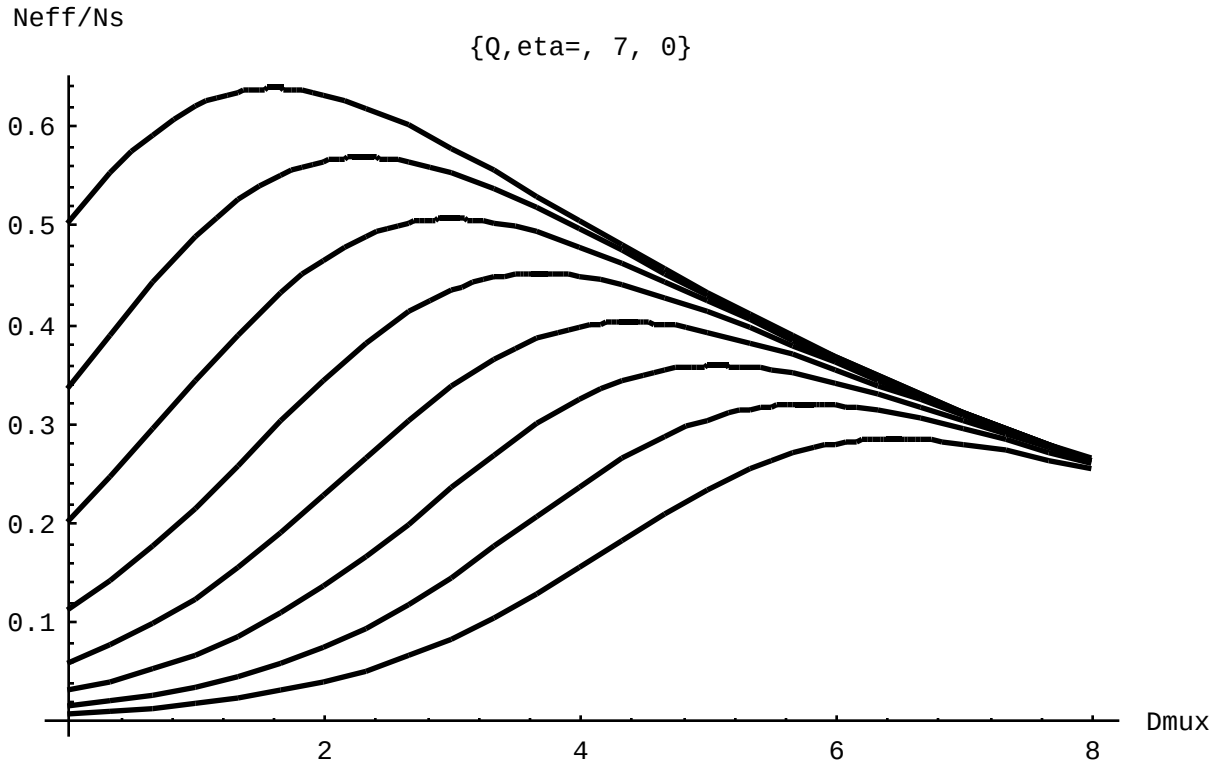
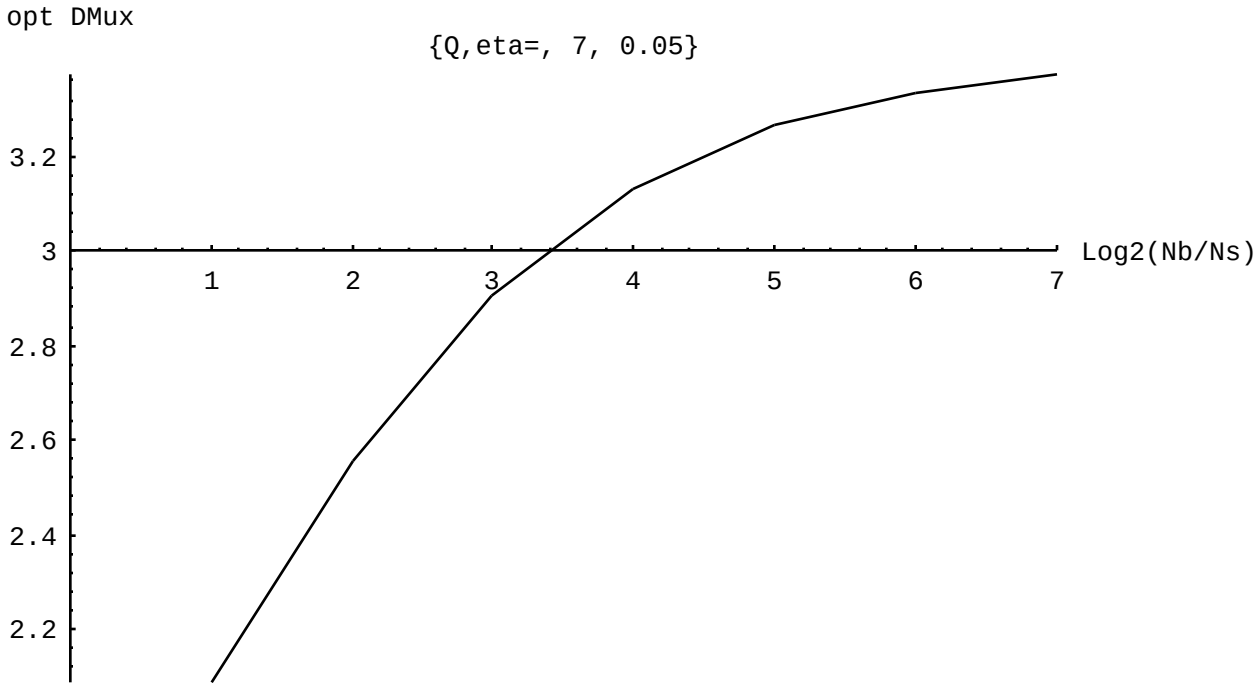
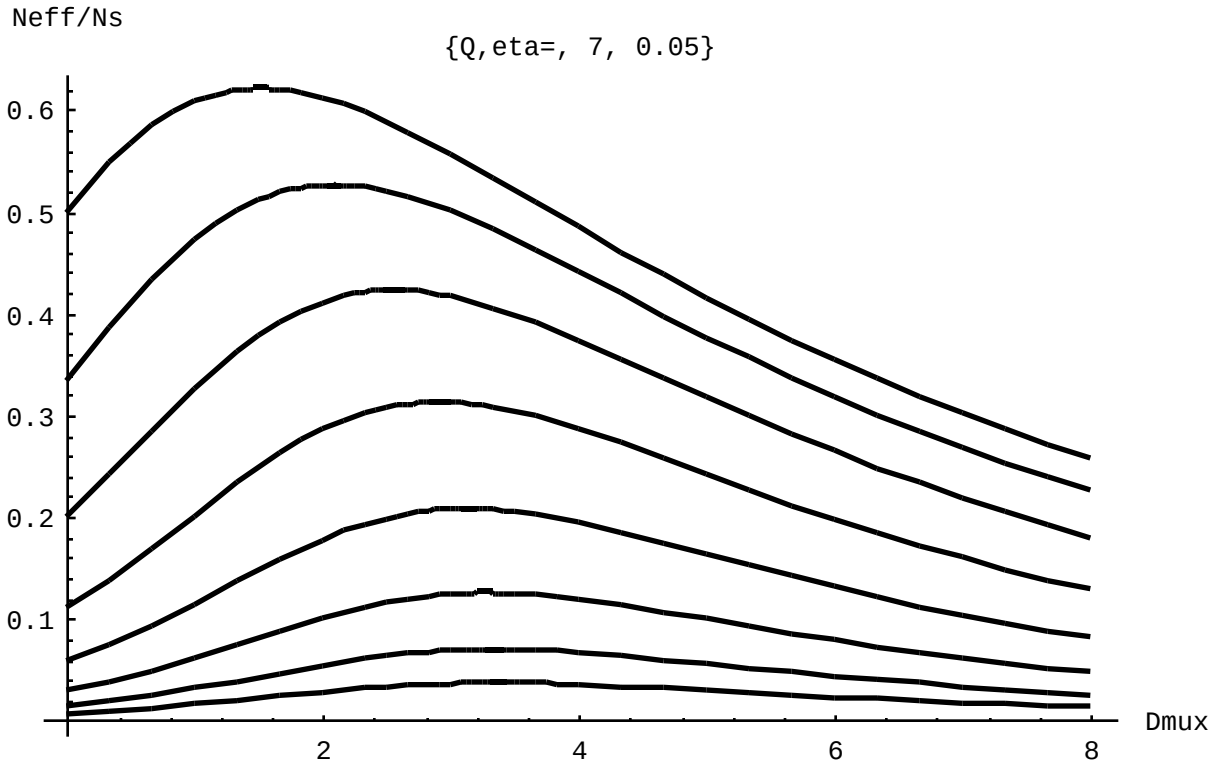
1
2
3
4
5
6
7
Log2(Nb/Ns)


Neff/Ns
{Q,eta=, 7, 0}
0.6
0.5
0.4
0.3
0.2
0.1
2
4
6
8
Dmux


opt DMux
{Q,eta=, 7, 0.05}
3.2
3
2.8
2.6
2.4
2.2
1
2
3
4
5
6
7
Log2(Nb/Ns)


Neff/Ns
{Q,eta=, 7, 0.05}
0.6
0.5
0.4
0.3
0.2
0.1
2
4
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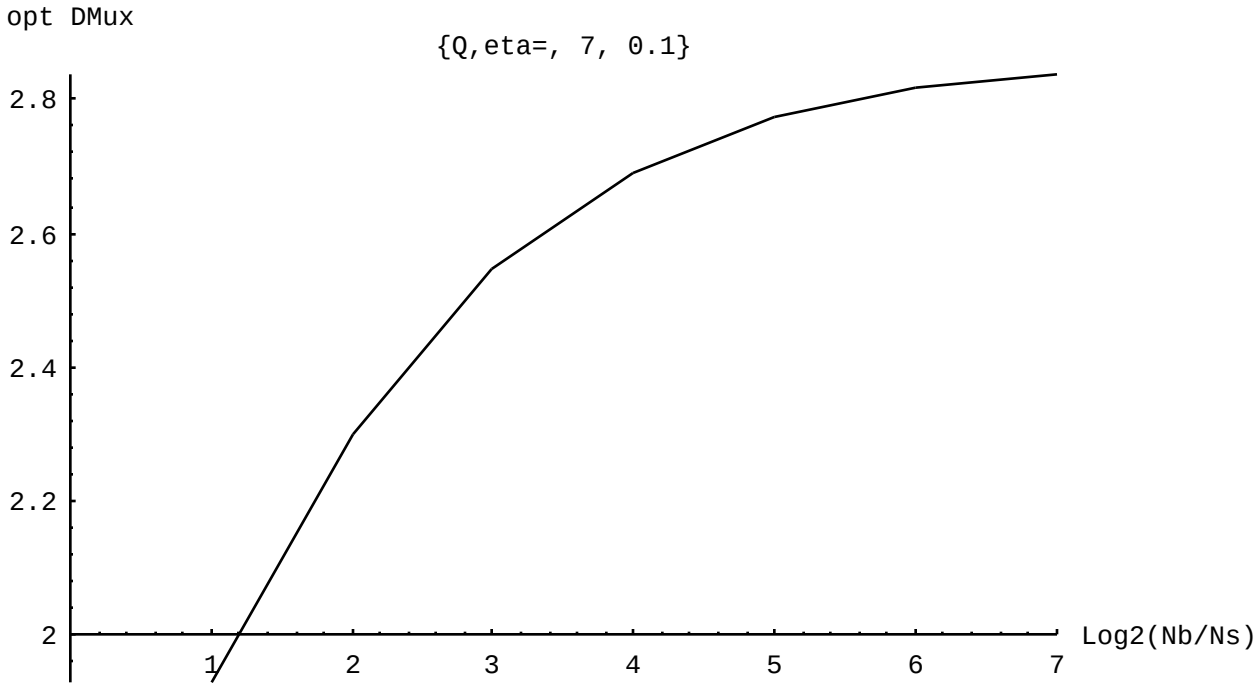
8
Dmux


opt DMux
{Q,eta=, 7, 0.1}
2.8
2.6
2.4
2.2
2
1
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3
4
5
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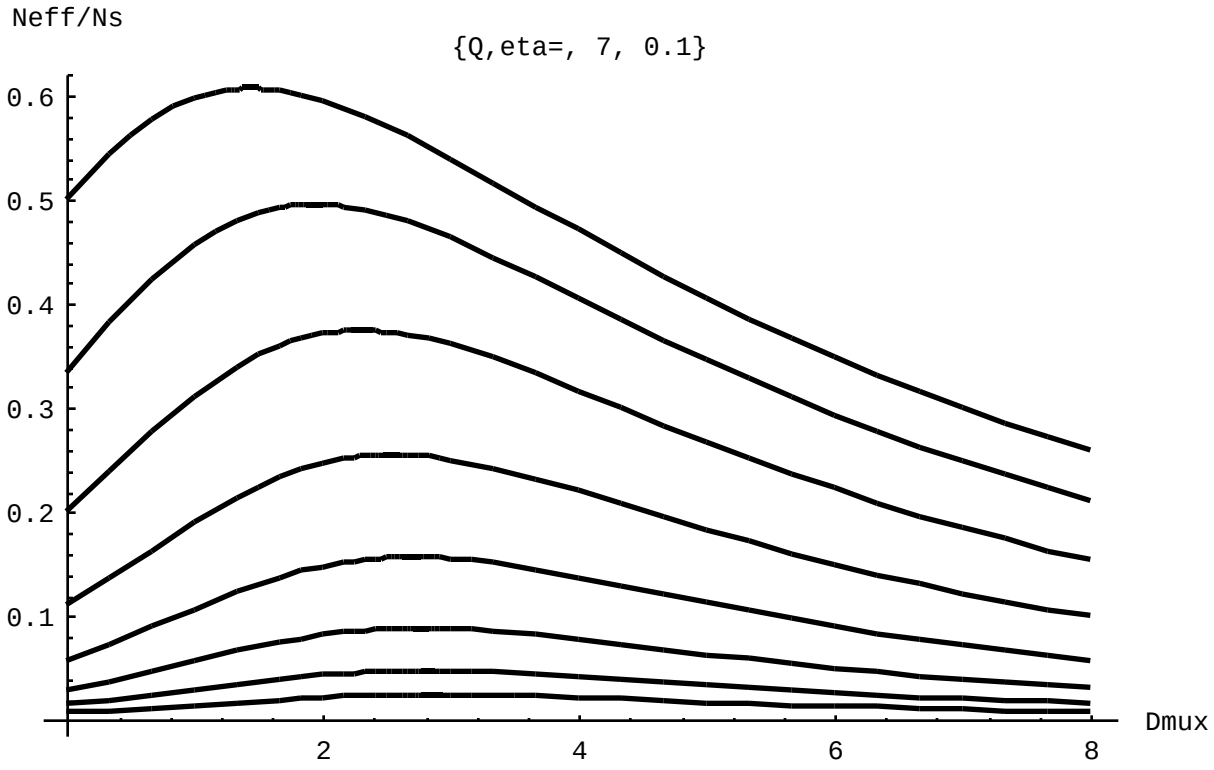
7
Log2(Nb/Ns)


Neff/Ns
{Q,eta=, 7, 0.1}
0.6
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0.4
0.3
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0.1
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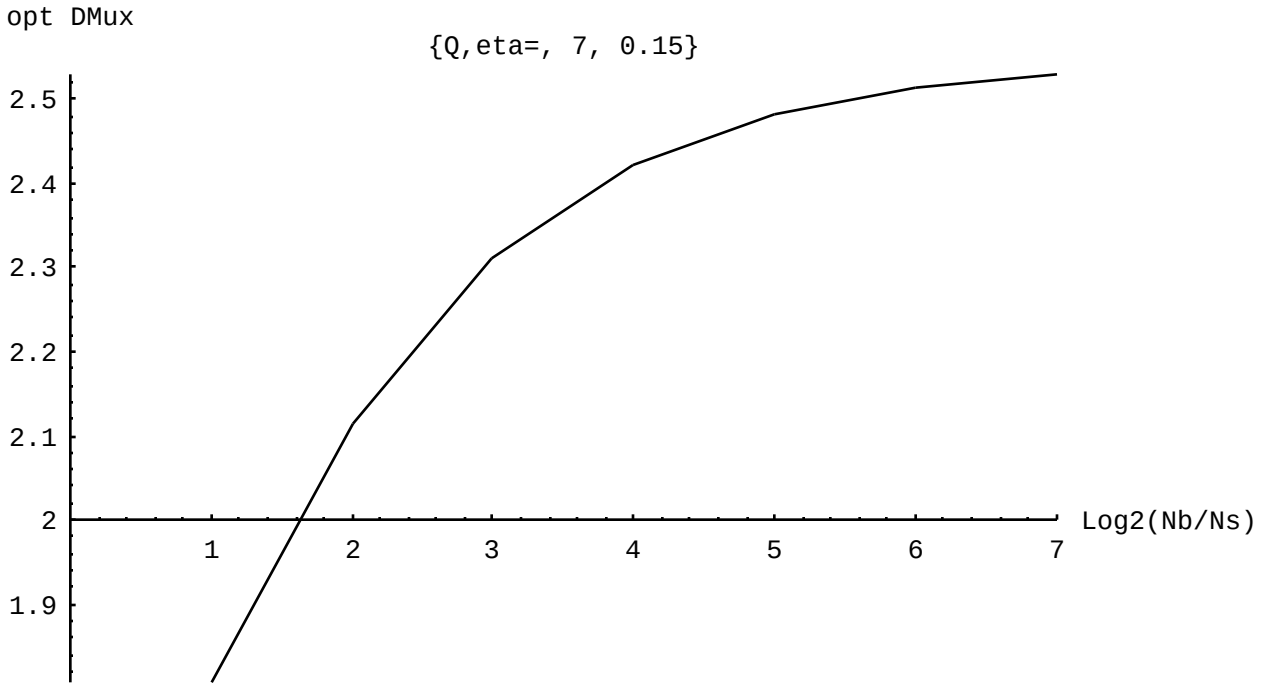
Dmux


opt DMux
{Q,eta=, 7, 0.15}
2.5
2.4
2.3
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2.1
2
1.9
1
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3
4
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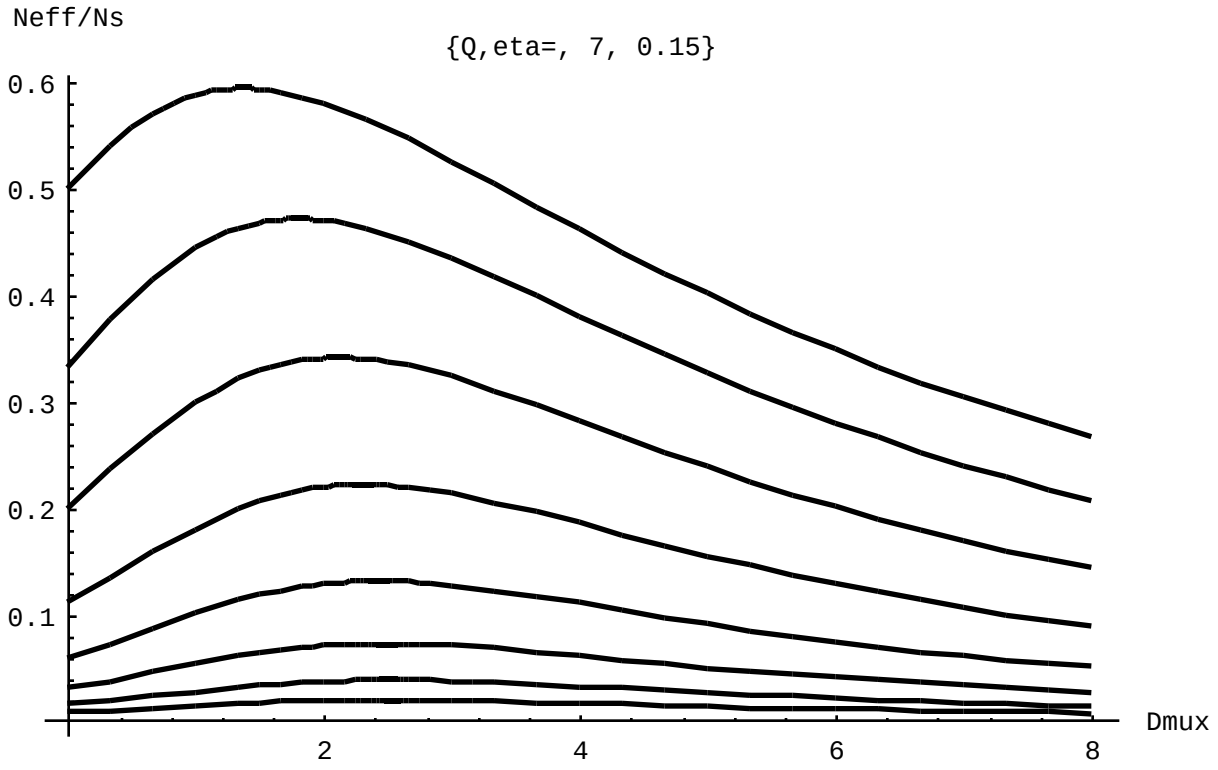
7
Log2(Nb/Ns)


Neff/Ns
{Q,eta=, 7, 0.15}
0.6
0.5
0.4
0.3
0.2
0.1
2
4
6
8
Dmux